Generalised teleparallel quintom dark energy non-minimally coupled with the scalar torsion and a boundary term

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Outline

1. Introduction to Teleparallel equivalent of general relativity
2. Teleparallel nonminimally coupled models
3. Teleparallel Quintom model
4. Conclusions

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices; Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

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Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^a{}_{\nu} = E_a{}^{\rho}(\partial_{\mu} e^a{}_{\nu} + w^a{}_{b\mu} e^b{}_{\nu})\,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

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Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu}\,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

$$R^{\rho}{}_{\theta\mu\nu}(\tilde{\Gamma}^{\alpha}{}_{\nu\mu}) = \partial_{\mu}\tilde{\Gamma}^{\rho}{}_{\theta\nu} - \partial_{\nu}\tilde{\Gamma}^{\rho}{}_{\theta\mu} + \tilde{\Gamma}^{\rho}{}_{\sigma\mu}\tilde{\Gamma}^{\sigma}{}_{\theta\nu} - \tilde{\Gamma}^{\rho}{}_{\sigma\nu}\tilde{\Gamma}^{\sigma}{}_{\theta\mu} = 0$$

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$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} \equiv 0\,.$$

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Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\rm TEGR} = \int \left[T + 2\kappa^2 L_{\rm m}\right] e\, d^4x \,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e^a_\mu) = \sqrt{-g}$, $L_{\rm m}$ matter Lagrangian and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

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Nonminimally coupled scalar fields with the scalar curvature

- One modification that it is important for our presentation is considering scalar fields nonminimally coupled with gravity. For example, a coupling with the scalar field ϕ to the scalar curvature R as follows (scalar-tensor theories)

$$S = \int \left[F(\phi) R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L_{\rm m} \right] \sqrt{-g}\, d^4 x \,.$$

  Here $V(\phi)$ is an energy potential energy.

- This action has the following theories:
  - $F = 1$: minimally coupled (GR+ scalar field).
  - $F = \phi$: Brans-Dicke theory (first scalar modified gravity).
  - $F = 1 + \phi^2$: nonminimally coupled with interesting applications in cosmology.

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Nonminimally coupled scalar fields to the scalar torsion

An alternative approach has been to consider a scalar field nonminimally coupled to torsion. The following action is considered

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Teleparallel quintessence with a nonminimal coupling to a boundary term

With the aim of unifying both of the previous considered approaches, we proposed a more general action given by (S. Bahamonde and M. Wright Phys. Rev. D **92** (2015) no.8, 084034); M. Zubair, S. Bahamonde and M. Jamil, Eur. Phys. J. C **77** (2017) no.7, 472)

$$S = \int \left[F(\phi)T + G(\phi)B + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + L_{\rm m} \right] e\, d^4x. \quad (1)$$

- This action was motivated by the relationship $R = -T + B$.
- Setting $F(\phi) = -G(\phi) \rightarrow$ nonminimal coupling to the scalar curvature R.
- Setting $G = 0 \rightarrow$ the same action as before: Teleparallel Dark Energy.
- $F = 1, G = \chi\phi^2$: late-time acceleration without fine tuning.

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Quintom model

- One can generalise this by having two scalar fields which leads to the following quintom models (M. Marciu, Phys. Rev. D **93** (2016) no.12, 123006)

$$S = \int \Big[\frac{R}{2} + \frac{1}{2}\Big(f_1(\phi) + f_2(\sigma)\Big)R + \frac{1}{2}\xi\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\chi\partial_\mu\sigma\partial^\mu\sigma - V(\phi,\sigma) + L_{\rm m}\Big]e\,d^4x\,,$$

  where now V depends on two scalar fields σ, ϕ.
- In Marciu 2016: $f_1 = -c_1\phi^2, f_2 = c_2\sigma^2, \chi = -\xi = 1$ (one phantom scalar field+one canonical).
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Our new theory: Generalised Quintom Teleparallel

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- Clearly, this model contains all the other previous theories mentioned (remember again $R = -T + B$).

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Cosmological equations

- Flat FRW cosmology $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ with tetrads $e^i_\mu = \mathrm{diag}(1, a(t), a(t), a(t))$. Moreover, consider a perfect fluid for matter.
- The modified FRW equations for this theory are:

$$\begin{aligned}
3H^2(1 + f_1(\phi) + f_2(\sigma)) &= \rho_{\rm m} + V(\phi,\sigma) + \frac{1}{2}\xi\dot{\phi}^2 \\
&\quad + \frac{1}{2}\chi\dot{\sigma}^2 + 3H(g_1'(\phi)\dot{\phi} + g_2'(\sigma)\dot{\sigma})\,, \\
(3H^2 + 2\dot{H})(1 + f_1(\phi) + f_2(\sigma)) &= -p_{\rm m} + V(\phi,\sigma) - \frac{1}{2}\xi\dot{\phi}^2 \\
&\quad - \frac{1}{2}\chi\dot{\sigma}^2 - 2H(\dot{\phi}f_1'(\phi) + \dot{\sigma}f_2'(\sigma)) \\
&\quad + \ddot{g}_1(\phi) + \ddot{g}_2(\sigma)\,.
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- Flat FRW cosmology $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ with tetrads $e^i_\mu = \mathrm{diag}(1, a(t), a(t), a(t))$. Moreover, consider a perfect fluid for matter.
- The modified FRW equations for this theory are:

$$\begin{aligned}
3H^2(1 + f_1(\phi) + f_2(\sigma)) &= \rho_{\rm m} + V(\phi,\sigma) + \frac{1}{2}\xi\dot{\phi}^2 \\
&\quad + \frac{1}{2}\chi\dot{\sigma}^2 + 3H(g_1'(\phi)\dot{\phi} + g_2'(\sigma)\dot{\sigma})\,, \\
(3H^2 + 2\dot{H})(1 + f_1(\phi) + f_2(\sigma)) &= -p_{\rm m} + V(\phi,\sigma) - \frac{1}{2}\xi\dot{\phi}^2 \\
&\quad - \frac{1}{2}\chi\dot{\sigma}^2 - 2H(\dot{\phi}f_1'(\phi) + \dot{\sigma}f_2'(\sigma)) \\
&\quad + \ddot{g}_1(\phi) + \ddot{g}_2(\sigma)\,.
\end{aligned}$$

Cosmological equations

- In addition, the scalar field equations:

$$\xi(\ddot{\phi}+3H\dot{\phi})+3H^2 f_1'(\phi)+3g_1'(\phi)\Big(3H^2+\dot{H}\Big)+\frac{V(\phi,\sigma)}{\partial\phi} = 0\,,$$
$$\chi(\ddot{\sigma}+3H\dot{\sigma})+3H^2 f_2'(\sigma)+3g_2'(\sigma)\Big(3H^2+\dot{H}\Big)+\frac{V(\phi,\sigma)}{\partial\sigma} = 0\,.$$

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Cosmology: asantz assumed

Let us now assume the following ansatz:

- $f_1(\phi) = c_1\phi^2\,, f_2(\sigma) = c_2\sigma^2\,, g_1(\phi) = c_3\phi^2\,, g_2(\sigma) = c_4\sigma^2$, where c_i $(i = 1, .., 4)$ are constants.
- $V(\phi, \sigma) = V_1(\phi) + V_2(\sigma) = V_1 e^{-\lambda_1\phi} + V_2 e^{-\lambda_2\phi}$, where V_1, V_2 and $\lambda_1 > 0, \lambda_2 > 0$ are constants.
- Barotropic fluid: $p_{\rm m} = (\gamma - 1)\rho_{\rm m}$, so we directly find that $\rho_{\rm m} = \rho_0 a(t)^{-3\gamma}$.
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Dynamical system

- Let us now introduce the dimensionless variables

$$s^2=\frac{\rho_{\rm m}}{3H^2}, x^2=\frac{\dot{\phi}^2}{6H^2}, y^2=\frac{V_1(\phi)}{3H^2}, z=2\sqrt{6}\xi\phi,$$
$$u^2=\frac{\dot{\sigma}^2}{6H^2}, v^2=\frac{V_2(\sigma)}{3H^2}, w=2\sqrt{6}\chi\sigma\,,$$

which straightforwardly generalise the normalised variables used to analyse standard quintessence (E. J. Copeland, A. R. Liddle and D. Wands,Phys. Rev. D **57** (1998) 4686)

- Then, the first FRW equation can be written as

$$s^2=1-\xi x^2-y^2+\frac{c_1}{24\xi^2}z^2-\frac{c_3}{\xi}xz-\chi u^2-v^2+\frac{c_2}{24\chi^2}w^2-\frac{c_4}{\chi}uw\,.$$

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- We can reduce the dimensionality of the dynamical system from 7 to 6.

Dynamical system

- By changing $N = \log a$, the 6D dynamical system becomes

$$\frac{dx}{dN} = \frac{1}{\sqrt{6}}\left(q - \sqrt{6}px\right), \quad (2)$$

$$\frac{dy}{dN} = -\frac{y}{2}\left(2p + \sqrt{6}\lambda_1 x\right), \quad (3)$$

$$\frac{dz}{dN} = 12\xi x, \quad (4)$$

$$\frac{du}{dN} = \frac{1}{\sqrt{6}}\left(q - \sqrt{6}pu\right), \quad (5)$$

$$\frac{dv}{dN} = -\frac{v}{2}\left(2p + \sqrt{6}\lambda_2 u\right), \quad (6)$$

$$\frac{dw}{dN} = 12u\chi. \quad (7)$$

Dynamical system

- We have defined the following quantities

$$
\begin{aligned}
p &= -\frac{1}{2\left(\chi^3\left(c_1\xi z^2+6c_3^2z^2+24\xi^3\right)+c_2\xi^3w^2\chi+6c_4^2\xi^3w^2\right)}\Big[3\chi^3\Big(c_1z(4c_3z\\
&+\xi(16\xi x+\gamma z))+4\Big(3c_3^2z^2-c_3\xi\left(6\xi x(4\xi x+(\gamma-2)z)+\sqrt{6}\lambda_1y^2z\right)\\
&-6\xi^3\Big(\gamma\left(u^2\chi+v^2+\xi x^2+y^2-1\right)-2\left(u^2\chi+\xi x^2\right)\Big)\Big)\Big)\\
&+c_2\xi^3w(4c_4w+\chi(16u\chi+\gamma w))+12c_4^2\xi^3w^2-4c_4\xi^3\chi\Big(6u\chi(4u\chi\\
&+(\gamma-2)w)+\sqrt{6}\lambda_2v^2w\Big)\Big],\\
q &= -\frac{\sqrt{6}c_1z+\sqrt{6}c_3(p+3)z+6\xi\left(\sqrt{6}\xi x-\lambda_1y^2\right)}{2\xi^2},\\
\lambda_1 &= -\frac{V_1'(\phi)}{V_1(\phi)},\\
\lambda_2 &= -\frac{V_2'(\sigma)}{V_2(\sigma)}.
\end{aligned}
$$

Dynamical system: analysis

- There are 21 critical points for the dynamical system, but only 13 satisfy $y \geq 0$ and $v \geq 0$ which ensures that the potentials are positive:

Point	x	y	z	u	v	w
O	0	0	0	0	0	0
$A_\pm$	0	0	0	0	0	$\pm 2\sqrt{-\frac{6}{c_2}}\kappa\chi$
$B_\pm$	0	0	$\pm 2\sqrt{-\frac{6}{c_1}}\kappa\xi$	0	0	0
$C_\pm$	0	0	0	0	$\frac{\sqrt{2(c_2+3c_4)}\sqrt{\chi(c_2+3c_4)\pm\Delta_1}}{\sqrt{c_2\chi}\lambda_2}$	$\frac{2\sqrt{6}(\chi(c_2+3c_4)\pm\Delta_1)}{c_2\lambda_2}$
$D_\pm$	0	$\frac{\sqrt{2(c_1+3c_3)}\sqrt{c_1\xi+3c_3\xi\pm\Delta_2}}{\sqrt{c_1\xi}\lambda_1}$	$\frac{2\sqrt{6}(c_1\xi+3c_3\xi\pm\Delta_2)}{c_1\lambda_1}$	0	0	0
$E_\pm$	0	$\sqrt{\frac{c_1+3c_3}{\sqrt{6}\lambda_1\xi}z}$	z	0	$\frac{\sqrt{c_2+3c_4}\sqrt{12c_2\lambda_1\xi^2\chi+36c_4\lambda_1\xi^2\chi\pm\Delta_3\sqrt{\lambda_1}\xi\chi}}{\sqrt{6\chi c_2\lambda_1}\lambda_2\xi}$	$\frac{\chi\left(12\sqrt{\lambda_1}\xi(c_2+3c_4)\pm\Delta_3\right)}{\sqrt{6\lambda_1}c_2\lambda_2\xi}$
$F_\pm$	0	$\sqrt{\frac{c_1+3c_3}{\sqrt{6}\lambda_1\xi}z}$	z	0	$-\frac{\sqrt{c_2+3c_4}\sqrt{12c_2\lambda_1\xi^2\chi+36c_4\lambda_1\xi^2\chi\pm\Delta_3\sqrt{\lambda_1}\xi\chi}}{\sqrt{6\chi c_2\lambda_1}\lambda_2\xi}$	$\frac{\chi\left(12\sqrt{\lambda_1}\xi(c_2+3c_4)\pm\Delta_3\right)}{\sqrt{6\lambda_1}c_2\lambda_2\xi}$

Dynamical system: analysis

- The point O is always a saddle point and it represents a matter dominated era.
- Critical points A_+ and A_- correspond to a dynamical scenario where the first quintom field ϕ is absent, whereas the second quintom field σ is frozen, without any kinetic or potential energy. DE dominated points.
- Critical points $B_\pm$ is similar as $A_\pm$ but the other scalar field is frozen.
- All the remaining points are non-hyperbolic, hence, linear stability fails! All those points represent dark energy dominated universe (see paper for more details).

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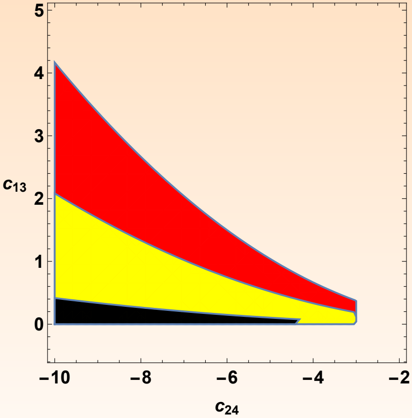

Figure: Regions where the points $A_{\pm}$ are stable. The black, yellow and red regions represent the cases where $\xi = 0.1, 0.5$ and 1 respectively. Here: $c_{24} = c_2/c_4$ and $c_{13} = c_1 - c_3 c_{24}$.

Numerical analysis: quintom coupled only with T

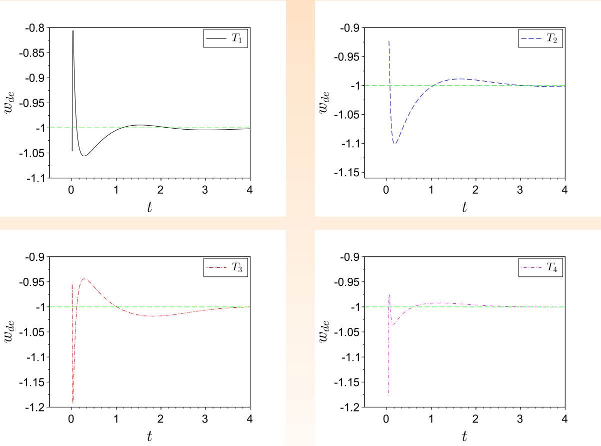

Figure: The evolution of the dark energy equation of state for scalar torsion coupling models T_1, T_2, T_3, T_4

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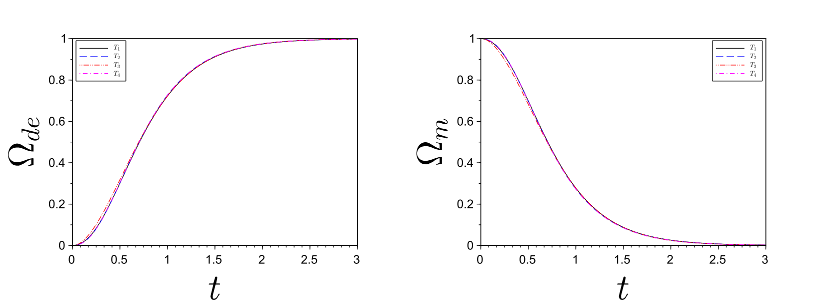

Figure: The evolution of the quintom energy density and matter energy density for scalar torsion coupling models

Conclusions

- Teleparallel gravity is al alternative (and equivalent than GR) formulation of gravity where curvature is zero but torsion is non trivial.
- We introduced a new quintom (two scalar field) nonminimally coupled model with both a torsion scalar T and a boundary term B (which is connected with scalar curvature via $R = -T + B$)
- We analysed the dynamical system which is a 6D one with 13 physical critical points. One critical point is a matter saddle point, the others energy dominated eras. We classified them and analysed (see paper for more details).
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