Teleparallel Gravity and its modifications

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30 May 2018

Outline

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

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Introducing TEGR

- In a gravitational theory which satisfies $\nabla_\lambda g_{\mu\nu} = 0$, one has that the curvature can be split as

Ricci's theorem

$$\bar{R}^{\lambda}{}_{\mu\sigma\nu} = R^{\lambda}{}_{\mu\sigma\nu} + \nabla_\sigma K_\nu{}^\lambda{}_\mu - \nabla_\nu K_\sigma{}^\lambda{}_\mu + K_\sigma{}^\lambda{}_\rho K_\nu{}^\rho{}_\mu - K_\sigma{}^\rho{}_\mu K_\nu{}^\lambda{}_\rho\,,$$

where $R^{\lambda}{}_{\mu\sigma\nu}$ is the curvature tensor computed with the Levi-Civita connection and $K^\rho{}_{\mu\nu} = \frac{1}{2}(T_\mu{}^\rho{}_\nu + T_\nu{}^\rho{}_\mu - T^\rho{}_{\mu\nu})$ is the contorsion tensor related to the torsion tensor $T_\mu{}^\rho{}_\nu$. ∇_μ is computed with respect to any connection.

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- By contracting the curvature tensor with the metric $g^{\mu\nu}\bar{R}^{\lambda}{}_{\mu\lambda\nu} \equiv \bar{R}$ (Ricci scalar), one gets

Ricci's theorem final

$$\bar{R} = R + T - B\,,$$

where $B = \frac{2}{e}\partial_{\mu}(eT^{\mu})$ is a boundary term in the action (see later) and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$ is the scalar torsion.

- Here $e = \sqrt{-g} = \det(e^{\mu}_{a})$ and $T_{\mu} = T^{\lambda}{}_{\lambda\mu}$.

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General Relativity case

- GR assumes no torsion, then $T^{\lambda}{}_{\mu\nu} = 0$, so $T = 0$ and then $B = 0$. Hence, $\bar{R} = R$ and then the Einstein-Hilbert action is constructed with it:

$$S_{\mathrm{GR}} = \int \left[R + 2\kappa^2 \mathcal{L}_{\mathrm{m}}\right] \sqrt{-g}\, d^4x \,.$$

- Here, $\kappa^2 = 8\pi G$ and $\mathcal{L}_{\mathrm{m}}$ is a matter Lagrangian.
- GR relies on the Levi-Civita connection which is torsionless and symmetric $\Gamma^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\mu\nu}$.
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Another fundamental theory?

Question

Is it possible to construct an alternative and equivalent gravitational theory different than GR?

Answer

Yes, it is possible, its name is Teleparallel equivalent of General Relativity, or in short, Teleparallel gravity.

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Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called “Weitzenböck connection”:

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_\mu e^a{}_\nu = E_a{}^{\rho}(\partial_\mu e^a{}_\nu + \omega^a{}_{b\mu} e^b{}_\nu)\,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}\,.$$

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Connection and curvature in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu}\,,$$

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

$$R^{\rho}{}_{\nu\mu\lambda} = \partial_{\mu}\Gamma^{\rho}{}_{\nu\lambda} - \partial_{\lambda}\Gamma^{\rho}{}_{\nu\mu} + \Gamma^{\rho}{}_{\mu\sigma}\Gamma^{\sigma}{}_{\nu\lambda} - \Gamma^{\rho}{}_{\lambda\sigma}\Gamma^{\sigma}{}_{\nu\mu} = 0\,.$$

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Teleparallel action

- Remember: $\bar{R} = R + T - B$, then we have that the full curvature is zero, $\bar{R} = 0$. This gives us a fundamental relationship:

Fundamental relationship

$$R = -T + B\,.$$

- The teleparallel action is formulated based on the torsion scalar T:

$$S_{\text{TEGR}} = \int \left[T + 2\kappa^2 \mathcal{L}_{\text{m}}\right] e\, d^4x\,.$$

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Two different ways of understanding gravity

- Then R and T differ by a boundary term in the action so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them. All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

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Two different ways of understanding gravity

Do not get confused with Einstein-Cartan gravity

Einstein-Cartan gravity is a theory where torsion and curvature are non-zero. This theory has more degrees of freedom than GR (and TEGR) and torsion is interpreted in a completely different way as in TEGR (related to spin)

Teleparallel gravity vs General Relativity

Two completely equivalent ways of understanding gravity:

Connections and strength fields

G.R. $\Longrightarrow$ Levi-Civita connection $\Longrightarrow$ Curvature with vanishing torsion
TEGR $\Longrightarrow$ Weitzenböck connection $\Longrightarrow$ Torsion with vanishing curvature (flat).

How gravity is explained in both theories?

GR $\Longrightarrow$ Geometry (curvature of space-time) $\Longrightarrow$ geodesic equations
TEGR $\Longrightarrow$ Forces $\Longrightarrow$ Force equations, similarly as Newtonian Eqs (no geodesic eq.).

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Gauge structure

GR $\Longrightarrow$ NO (only diffeomorphism)
TEGR $\Longrightarrow$ Gauge theory of the translations

Must have the equivalence principle?

GR $\Longrightarrow$ YES
TEGR $\Longrightarrow$ Can survive with or without

Can we separate inertia with gravity?

GR $\Longrightarrow$ NO (mixed) $\Longrightarrow$ No tensorial expression for the gravitational energy-momentum density
TEGR $\Longrightarrow$ YES $\Longrightarrow$ gravitational energy-momentum density is a tensor.

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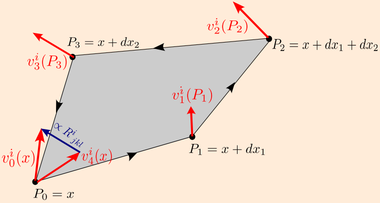
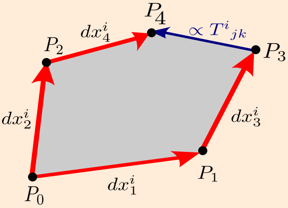
Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported





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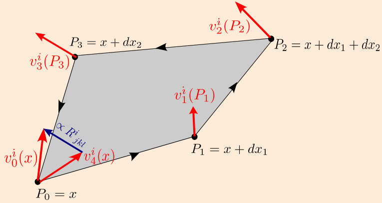

Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

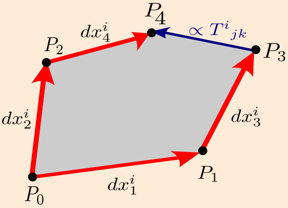

Figure: Open parallelogram

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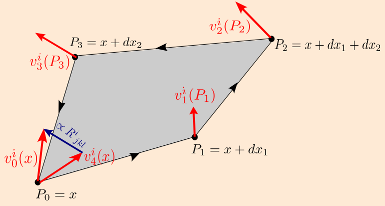

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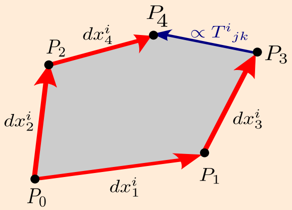

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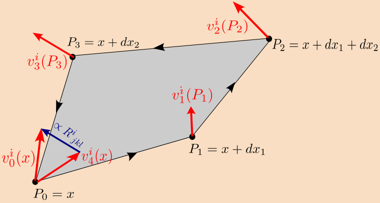

Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

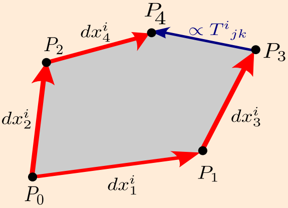

Figure: Open parallelogram

Teleparallel gravity vs General Relativity

Geometrical differences

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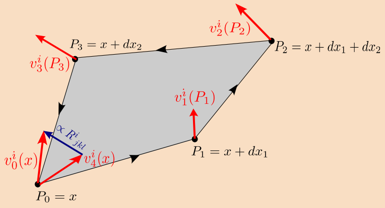

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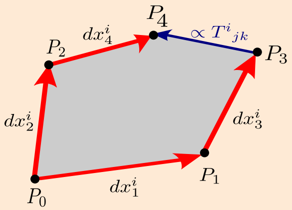

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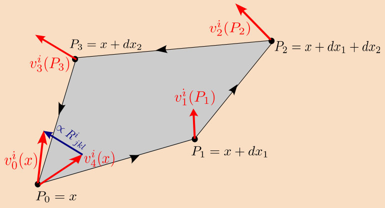

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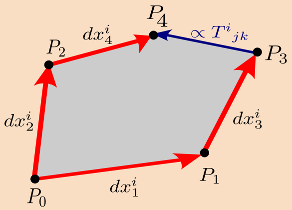

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Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

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$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \,.$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
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$f(T)$ gravity: Cosmology

- In flat FLRW space-time $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, one good tetrad is $e^\mu_a = \mathrm{diag}(1, a(t), a(t), a(t))$.
- The $T + f(T)$ modified FLRW equations with standard perfect fluid characterised by ρ and p are

$$3H^2 + 6H^2 f_T + \frac{1}{2} f(T) = \kappa^2 \rho \,,$$

$$3H^2 + 2\dot{H} + 2(3H^2 + \dot{H}) f_T + 2H \dot{f}_T + \frac{1}{2} f(T) = -\kappa^2 p \,,$$

where $f_T = df(T)/dT$, $\kappa^2 = 8\pi G$ and $H = \dot{a}/a$. The scalar torsion is $T = -6H^2$.
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- In GR (ΛCDM), dark energy is represented by a cosmological constant which acts as a non-standard matter (exotic) which violates the energy conditions (repulsive gravitational force).
- In $f(T)$ gravity, it is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.
- We avoid the issue about non-standard matter but the price to pay is that our theory is more difficult than GR or TEGR (1 extra degree of freedom).
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- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
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Extending $f(T)$ gravity

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Extending $f(T)$ gravity

Interesting remark 1

It is always possible to reconstruct the modifications of General Relativity by adding the boundary term B in the action. So, it is possible to obtain the Teleparallel equivalent of any theory based on R (for example scalar tensor theories, or Gauss-Bonnet) by adding this term (or other boundary terms).

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- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero cuvature and non-zero torsion (Weitzbröck connection).
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