Teleparallel Gravity and its modifications

Sebastián Bahamonde

PhD student at Department of Mathematics, University College London.

IMPRS on Gravitational Waves, Max Planck Research School, Lüneburger Heide

15 March 2018

Outline

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
  Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
  Latin letters $\rightarrow$ tangent space indices; ${E_a}^{\mu}$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^{\mu}{e^n}_{\mu} = \delta^n_m$ and ${E_m}^{\nu}{e^m}_{\mu} = \delta^{\nu}_{\mu}$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab}{e^a}_{\mu}{e^b}_{\nu}$

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
  Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_\mu e^a{}_\nu = E_a{}^{\rho}(\partial_\mu e^a{}_\nu + w^a{}_{b\mu} e^b{}_\nu)\,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}\,.$$

Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_\mu e^a{}_\nu = E_a{}^{\rho}(\partial_\mu e^a{}_\nu + w^a{}_{b\mu} e^b{}_\nu)\,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}\,.$$

Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called “Weitzenböck connection”:

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^a{}_{\nu} = E_a{}^{\rho} (\partial_{\mu} e^a{}_{\nu} + w^a{}_{b\mu} e^b{}_{\nu}) \,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu} \,.$$

Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_\mu e^a{}_\nu = E_a{}^{\rho}(\partial_\mu e^a{}_\nu + w^a{}_{b\mu} e^b{}_\nu)\,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}\,.$$

Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called “Weitzenböck connection”:

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^a{}_{\nu} = E_a{}^{\rho} (\partial_{\mu} e^a{}_{\nu} + w^a{}_{b\mu} e^b{}_{\nu}) \,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu} \,.$$

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu} \,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu} \,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu}\,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu} \,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu} \,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} \equiv 0 \,.$$

Connection in TEGR

- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu}\,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} \equiv 0\,.$$

Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\rm TEGR} = \int \left[-T + 2\kappa^2 \mathcal{L}_{\rm m}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e^{\mu}_{a}) = \sqrt{-g}$, $\mathcal{L}_{\rm m}$ matter Lagrangian and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\text{TEGR}} = \int \left[-T + 2\kappa^2 \mathcal{L}_{\text{m}}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e^{\mu}_{a}) = \sqrt{-g}$, $\mathcal{L}_{\text{m}}$ matter Lagrangian and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\rm TEGR} = \int \left[-T + 2\kappa^2 \mathcal{L}_{\rm m}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e_a^\mu) = \sqrt{-g}$, $\mathcal{L}_{\rm m}$ matter Lagrangian and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

Two different ways of understanding gravity

Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them. All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

Do not get confused with Einstein-Cartan gravity

Einstein-Cartan gravity is a theory where torsion and curvature are non-zero. This theory has more degrees of freedom than GR (and TEGR) and torsion is interpreted in a completely different way as in TEGR (related to spin)

Two different ways of understanding gravity

Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them. All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

Do not get confused with Einstein-Cartan gravity

Einstein-Cartan gravity is a theory where torsion and curvature are non-zero. This theory has more degrees of freedom than GR (and TEGR) and torsion is interpreted in a completely different way as in TEGR (related to spin)

Teleparallel gravity vs General Relativity

Two completely equivalent ways of understanding gravity:

Connections and strength fields

G.R. $\Longrightarrow$ Levi-Civita connection $\Longrightarrow$ Curvature with vanishing torsion
TEGR $\Longrightarrow$ Weitzenböck connection $\Longrightarrow$ Torsion with vanishing curvature (flat).

How gravity is explained in both theories?

GR $\Longrightarrow$ Geometry (curvature of space-time) $\Longrightarrow$ geodesic equations
TEGR $\Longrightarrow$ Forces $\Longrightarrow$ Force equations, similarly as Newtonian Eqs (no geodesic eq.).

Teleparallel gravity vs General Relativity

Two completely equivalent ways of understanding gravity:

Connections and strength fields

G.R. $\Longrightarrow$ Levi-Civita connection $\Longrightarrow$ Curvature with vanishing torsion
TEGR $\Longrightarrow$ Weitzenböck connection $\Longrightarrow$ Torsion with vanishing curvature (flat).

How gravity is explained in both theories?

GR $\Longrightarrow$ Geometry (curvature of space-time) $\Longrightarrow$ geodesic equations
TEGR $\Longrightarrow$ Forces $\Longrightarrow$ Force equations, similarly as Newtonian Eqs (no geodesic eq.).

Teleparallel gravity vs General Relativity

Gauge structure

GR $\Longrightarrow$ NO (only diffeomorphism)
TEGR $\Longrightarrow$ Gauge theory of the translations

Must have the equivalence principle?

GR $\Longrightarrow$ YES
TEGR $\Longrightarrow$ Can survive with or without

Can we separate inertia with gravity?

GR $\Longrightarrow$ NO (mixed) $\Longrightarrow$ No tensorial expression for the gravitational energy-momentum density
TEGR $\Longrightarrow$ YES $\Longrightarrow$ gravitational energy-momentum density is a tensor.

Teleparallel gravity vs General Relativity

Gauge structure

GR $\Longrightarrow$ NO (only diffeomorphism)
TEGR $\Longrightarrow$ Gauge theory of the translations

Must have the equivalence principle?

GR $\Longrightarrow$ YES
TEGR $\Longrightarrow$ Can survive with or without

Can we separate inertia with gravity?

GR $\Longrightarrow$ NO (mixed) $\Longrightarrow$ No tensorial expression for the gravitational energy-momentum density
TEGR $\Longrightarrow$ YES $\Longrightarrow$ gravitational energy-momentum density is a tensor.

Teleparallel gravity vs General Relativity

Gauge structure

GR $\Longrightarrow$ NO (only diffeomorphism)
TEGR $\Longrightarrow$ Gauge theory of the translations

Must have the equivalence principle?

GR $\Longrightarrow$ YES
TEGR $\Longrightarrow$ Can survive with or without

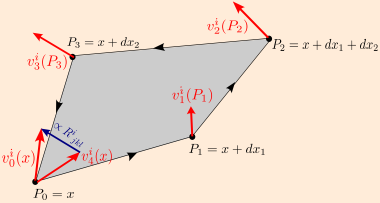
Can we separate inertia with gravity?

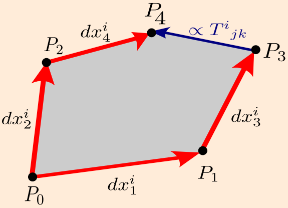
GR $\Longrightarrow$ NO (mixed) $\Longrightarrow$ No tensorial expression for the gravitational energy-momentum density
TEGR $\Longrightarrow$ YES $\Longrightarrow$ gravitational energy-momentum density is a tensor.

Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported





Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported

Curvature

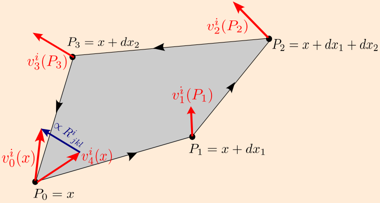


Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

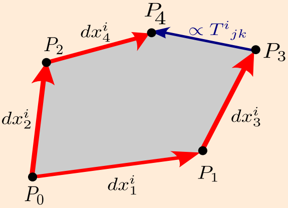


Figure: Open parallelogram

Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported

Curvature

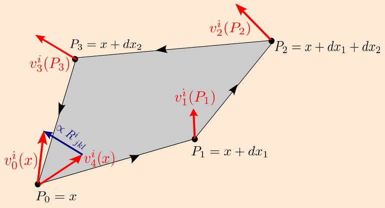


Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

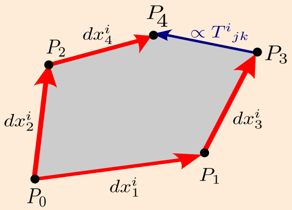


Figure: Open parallelogram

Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.

Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported

Curvature

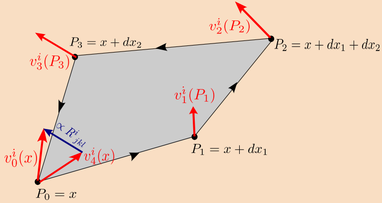


Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

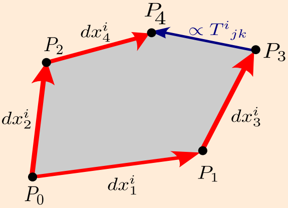


Figure: Open parallelogram

Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported

Curvature

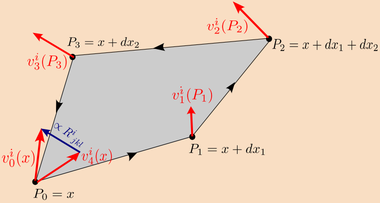


Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

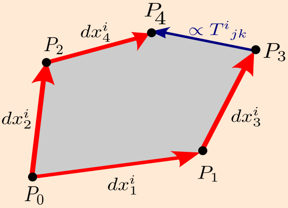


Figure: Open parallelogram

Teleparallel gravity vs General Relativity

Geometrical differences

Curvature $\Longrightarrow$ how the tangent spaces roll along the curve.
Torsion $\Longrightarrow$ how tangent spaces twist about a curve when they are parallel transported

Curvature

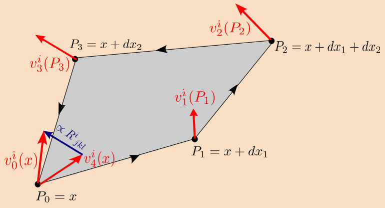


Figure: Transporting a vector in a closed trajectory creates a different vector

Torsion

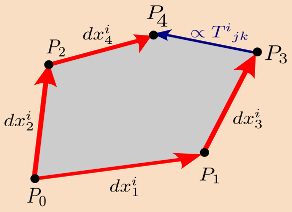


Figure: Open parallelogram

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume $w^a{}_{bc} = 0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad ($w^a{}_{bc} = 0$) but choosing "good tetrads".

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume $w^a{}_{bc} = 0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad ($w^a{}_{bc} = 0$) but choosing "good tetrads".

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume $w^a{}_{bc} = 0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad ($w^a{}_{bc} = 0$) but choosing "good tetrads".

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume ${w^a}_{bc}=0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad (${w^a}_{bc}=0$) but choosing "good tetrads".

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find ${w^a}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume $w^a{}_{bc} = 0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad ($w^a{}_{bc} = 0$) but choosing "good tetrads".

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
- Mostly all the literature use (2).

Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If ones modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories?

1. Do NOT assume ${w^a}_{bc} = 0$, (like covariant $f(T)$ gravity)
2. Use pure tetrad (${w^a}_{bc} = 0$) but choosing “good tetrads”.

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find ${w^a}_{bc}$.
- Mostly all the literature use (2).

$f(T)$ gravity

- In analogy with $f(R)$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(R)$

$f(T)$ gravity

- In analogy with $f(R)$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\to$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow$ $f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T)$ $\neq$ Field equations of $f(R)$

$f(T)$ gravity

- In analogy with $f(R)$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \,.$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow$ $f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(R)$

$f(T)$ gravity

- In analogy with $f(R)$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow$ $f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(R)$

$f(T)$ gravity

- In analogy with $f(R)$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory.**
- T is not invariant under local LT $\Longrightarrow f(T)$ **is also not invariant under local LT.**

Not equivalency between $f(R)$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(R)$

$f(T)$ gravity: Cosmology

- In flat FLRW space-time $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, one good tetrad is $e_a^\mu = \mathrm{diag}(1, a(t), a(t), a(t))$.
- The $T + f(T)$ modified FLRW equations with standard perfect fluid characterised by ρ and p are

$$3H^2 + 6H^2 f_T + \frac{1}{2}f(T) = \kappa^2\rho\,,$$

$$3H^2 + 2\dot{H} + 2(3H^2 + \dot{H})f_T + 2H\dot{f}_T + \frac{1}{2}f(T) = -\kappa^2 p\,,$$

  where $f_T = df(T)/dT$, $\kappa^2 = 8\pi G$ and $H = \dot{a}/a$. The scalar torsion is $T = -6H^2$.
- One can think that these equations are standard FLRW + new terms coming from $f(T)$.
- The standard conservation equation also holds: $\dot{\rho} + 3H(\rho + p) = 0$

$f(T)$ gravity: Cosmology

- In flat FLRW space-time $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, one good tetrad is $e_a^\mu = \mathrm{diag}(1, a(t), a(t), a(t))$.
- The $T + f(T)$ modified FLRW equations with standard perfect fluid characterised by ρ and p are

$$3H^2 + 6H^2 f_T + \frac{1}{2} f(T) = \kappa^2 \rho \,,$$

$$3H^2 + 2\dot{H} + 2(3H^2 + \dot{H}) f_T + 2H \dot{f}_T + \frac{1}{2} f(T) = -\kappa^2 p \,,$$

where $f_T = df(T)/dT$, $\kappa^2 = 8\pi G$ and $H = \dot{a}/a$. The scalar torsion is $T = -6H^2$.

- One can think that these equations are standard FLRW + new terms coming from $f(T)$.
- The standard conservation equation also holds: $\dot{\rho} + 3H(\rho + p) = 0$

$f(T)$ gravity: Cosmology

- In flat FLRW space-time $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, one good tetrad is $e^{\mu}_{a} = \mathrm{diag}(1, a(t), a(t), a(t))$.
- The $T + f(T)$ modified FLRW equations with standard perfect fluid characterised by ρ and p are

$$3H^2 + 6H^2 f_T + \frac{1}{2} f(T) = \kappa^2 \rho \,,$$

$$3H^2 + 2\dot{H} + 2(3H^2 + \dot{H}) f_T + 2H \dot{f}_T + \frac{1}{2} f(T) = -\kappa^2 p \,,$$

where $f_T = df(T)/dT$, $\kappa^2 = 8\pi G$ and $H = \dot{a}/a$. The scalar torsion is $T = -6H^2$.

- One can think that these equations are standard FLRW + new terms coming from $f(T)$.
- The standard conservation equation also holds: $\dot{\rho} + 3H(\rho + p) = 0$

$f(T)$ gravity: Cosmology

- In flat FLRW space-time $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, one good tetrad is $e_a^\mu = \mathrm{diag}(1, a(t), a(t), a(t))$.
- The $T + f(T)$ modified FLRW equations with standard perfect fluid characterised by ρ and p are

$$3H^2 + 6H^2 f_T + \frac{1}{2} f(T) = \kappa^2 \rho \,,$$
$$3H^2 + 2\dot{H} + 2(3H^2 + \dot{H}) f_T + 2H \dot{f}_T + \frac{1}{2} f(T) = -\kappa^2 p \,,$$

  where $f_T = df(T)/dT$, $\kappa^2 = 8\pi G$ and $H = \dot{a}/a$. The scalar torsion is $T = -6H^2$.
- One can think that these equations are standard FLRW + new terms coming from $f(T)$.
- The standard conservation equation also holds: $\dot{\rho} + 3H(\rho + p) = 0$

$f(T)$ gravity: Cosmology

- In GR (ΛCDM), dark energy is represented by a cosmological constant which acts as a non-standard matter (exotic) which violates the energy conditions (repulsive gravitational force).
- In $f(T)$ gravity, it is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.
- We avoid the issue about non-standard matter but the price to pay is that our theory is more difficult than GR or TEGR (1 extra degree of freedom).
- It is consistent with CMB observations. Moreover, the $f(T)$ gravity model does not show tension on the Hubble constant that prevails in the ΛCDM cosmology

$f(T)$ gravity: Cosmology

- In GR (ΛCDM), dark energy is represented by a cosmological constant which acts as a non-standard matter (exotic) which violates the energy conditions (repulsive gravitational force).
- In $f(T)$ gravity, it is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.
- We avoid the issue about non-standard matter but the price to pay is that our theory is more difficult than GR or TEGR (1 extra degree of freedom).
- It is consistent with CMB observations. Moreover, the $f(T)$ gravity model does not show tension on the Hubble constant that prevails in the ΛCDM cosmology

$f(T)$ gravity: Cosmology

- In GR (ΛCDM), dark energy is represented by a cosmological constant which acts as a non-standard matter (exotic) which violates the energy conditions (repulsive gravitational force).
- In $f(T)$ gravity, it is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.
- We avoid the issue about non-standard matter but the price to pay is that our theory is more difficult than GR or TEGR (1 extra degree of freedom).
- It is consistent with CMB observations. Moreover, the $f(T)$ gravity model does not show tension on the Hubble constant that prevails in the ΛCDM cosmology

$f(T)$ gravity: Cosmology

- In GR (ΛCDM), dark energy is represented by a cosmological constant which acts as a non-standard matter (exotic) which violates the energy conditions (repulsive gravitational force).
- In $f(T)$ gravity, it is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.
- We avoid the issue about non-standard matter but the price to pay is that our theory is more difficult than GR or TEGR (1 extra degree of freedom).
- It is consistent with CMB observations. Moreover, the $f(T)$ gravity model does not show tension on the Hubble constant that prevails in the ΛCDM cosmology

$f(T)$ gravity: Cosmology and more

- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
- Bounce solutions: the universe governed by $f(T)$ gravity could be smooth and non-singular throughout the whole cosmic evolution (initial singularity can be solved)
- No further gravitational modes in $f(T)$ gravity
- One extra degree of freedom

$f(T)$ gravity: Cosmology and more

- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
- Bounce solutions: the universe governed by $f(T)$ gravity could be smooth and non-singular throughout the whole cosmic evolution (initial singularity can be solved)
- No further gravitational modes in $f(T)$ gravity
- One extra degree of freedom

$f(T)$ gravity: Cosmology and more

- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
- Bounce solutions: the universe governed by $f(T)$ gravity could be smooth and non-singular throughout the whole cosmic evolution (initial singularity can be solved)
- No further gravitational modes in $f(T)$ gravity
- One extra degree of freedom

$f(T)$ gravity: Cosmology and more

- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
- Bounce solutions: the universe governed by $f(T)$ gravity could be smooth and non-singular throughout the whole cosmic evolution (initial singularity can be solved)
- No further gravitational modes in $f(T)$ gravity
- One extra degree of freedom

$f(T)$ gravity: Cosmology and more

- Phantom-divide crossing.
- Sufficiently long accelerating phase of the Universe at early times can be naturally achieved, without the need of introducing an inflaton field
- Bounce solutions: the universe governed by $f(T)$ gravity could be smooth and non-singular throughout the whole cosmic evolution (initial singularity can be solved)
- No further gravitational modes in $f(T)$ gravity
- One extra degree of freedom

Conclusions

- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero cuvature and non-zero torsion (Weitzbröck connection).
- TG is equivalent on its field equations to GR, but their physical interpretation are different.
- Two alternative ways of understanding gravity: either GR (torsion zero and curvature non-zero) or TG (curvature zero and torsion non-zero).

Conclusions

- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero cuvature and non-zero torsion (Weitzbröck connection).
- TG is equivalent on its field equations to GR, but their physical interpretation are different.
- Two alternative ways of understanding gravity: either GR (torsion zero and curvature non-zero) or TG (curvature zero and torsion non-zero).

Conclusions

- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero cuvature and non-zero torsion (Weitzbröck connection).
- TG is equivalent on its field equations to GR, but their physical interpretation are different.
- Two alternative ways of understanding gravity: either GR (torsion zero and curvature non-zero) or TG (curvature zero and torsion non-zero).

Conclusions

- Similarly as in GR, it is possible to formulate modifications of teleparallel gravity to try to solve some well-known problems (singularities, dark energy, inflation, etc.)
- The most popular modified teleparallel gravity is $f(T)$ gravity that can mimic very well the cosmological data for the history of the Universe.
- There are other modifications such as teleparallel scalar tensor theories or considering other higher derivatives terms coming from torsion.

Conclusions

- Similarly as in GR, it is possible to formulate modifications of teleparallel gravity to try to solve some well-known problems (singularities, dark energy, inflation, etc.)
- The most popular modified teleparallel gravity is $f(T)$ gravity that can mimic very well the cosmological data for the history of the Universe.
- There are other modifications such as teleparallel scalar tensor theories or considering other higher derivatives terms coming from torsion.

Conclusions

- Similarly as in GR, it is possible to formulate modifications of teleparallel gravity to try to solve some well-known problems (singularities, dark energy, inflation, etc.)
- The most popular modified teleparallel gravity is $f(T)$ gravity that can mimic very well the cosmological data for the history of the Universe.
- There are other modifications such as teleparallel scalar tensor theories or considering other higher derivatives terms coming from torsion.