Black holes solutions in metric-affine gravity with dynamical torsion and nonmetricity

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Yukawa International Seminar 2022a Gravity, 15/02/2022

Based on JCAP **09** (2020), 057, Eur.Phys.J.C **81** (2021) 6, 495; JCAP **01** (2022) no.01, 011; arXiv:2201.10532 [gr-qc]; Jointly with Jorge Gigante Valcarcel.

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Outline

1. Brief introduction to Metric-affine gravity
   - Basic quantities
   - Dynamics
2. MAG models with dynamical torsion and nonmetricity
   - Spherical symmetry
   - Axial symmetry

Fundamental variables and characteristic tensors

- In the most general metric-affine setting, the fundamental variables are a **metric** $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\tilde{\Gamma}^{\rho}{}_{\mu\nu}$ (64 comp.) of an **affine connection.**

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Curvature	$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = \partial_{\rho}\tilde{\Gamma}^{\mu}{}_{\nu\sigma} - \partial_{\sigma}\tilde{\Gamma}^{\mu}{}_{\nu\rho} + \tilde{\Gamma}^{\mu}{}_{\tau\rho}\tilde{\Gamma}^{\tau}{}_{\nu\sigma} - \tilde{\Gamma}^{\mu}{}_{\tau\sigma}\tilde{\Gamma}^{\tau}{}_{\nu\rho}$
Torsion	$\tilde{T}^{\mu}{}_{\nu\rho} = \tilde{\Gamma}^{\mu}{}_{\nu\rho} - \tilde{\Gamma}^{\mu}{}_{\rho\nu}$
Nonmetricity	$\tilde{Q}_{\mu\nu\rho} = \tilde{\nabla}_{\mu}g_{\nu\rho} = \partial_{\mu}g_{\nu\rho} - \tilde{\Gamma}^{\sigma}{}_{\nu\mu}g_{\sigma\rho} - \tilde{\Gamma}^{\sigma}{}_{\rho\mu}g_{\nu\sigma}$

Dynamics of metric-affine geometry

- Gravitational action with dynamical torsion and nonmetricity:

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_m - \frac{1}{16\pi}\mathcal{L}_g(\tilde{\mathcal{R}},\mathcal{T},\mathcal{Q})\right]. \tag{2}$$

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- Correspondence between geometry and matter:

$$\frac{\delta S_g}{\delta e^a{}_\nu} = 16\pi\theta_a{}^\nu , \tag{3}$$

$$\frac{\delta S_g}{\delta \omega^a{}_{b\nu}} = 16\pi\Delta_a{}^{b\nu} . \tag{4}$$

Here $\theta_a{}^\nu$ is the energy-momentum tensor (canonical) and $\Delta_a{}^{b\nu}$ is the hypermomentum density tensor.

MAG models with dynamical torsion and nonmetricity

- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry ($Q_{\lambda\mu\nu} = g_{\mu\nu}W_\lambda$)

$$S = \int d^4x\sqrt{-g}\Big\{\mathcal{L}_{\rm m} + \frac{1}{64\pi}\Big[-4R - 6d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\mu[\lambda\nu\rho]} + 8\,d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{[\mu\nu]} + \frac{1}{8}\left(32e_1 + 8e_2 + 17d_1\right)\tilde{R}^{\lambda}{}_{\lambda\mu\nu}\tilde{R}^{\rho}{}_{\rho}{}^{\mu\nu} - 7d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{\lambda}{}_{\lambda}{}^{\mu\nu} + 3\left(1-2a_2\right)T_{[\lambda\mu\nu]}T^{[\lambda\mu\nu]}\Big]\Big\}\,.$$

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- Absence of a general Birkhoff's theorem in MAG: new spherically and axially symmetric vacuum solutions with independent dynamical torsion and nonmetricity fields[1,2].

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Spherical symmetry

- Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2+\#8+\#2=\#12$):

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^{\lambda}{}_{\mu\nu} = \mathcal{L}_\xi W_\mu = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda\rho\mu\nu} = 0$$

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- By solving these equations we find that torsion and nonmetricity behave as

$$T^t{}_{tr} = a(r)\,, \quad T^r{}_{tr} = b(r)\,, \quad T^{\theta_k}{}_{t\theta_k} = f(r)\,, \quad T^{\theta_k}{}_{r\theta_k} = g(r)$$
$$T^{\theta_k}{}_{t\theta_l} = e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, d(r)\,, \quad T^{\theta_k}{}_{r\theta_l} = e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, h(r)\,,$$
$$T^t{}_{\theta_k\theta_l} = \epsilon_{kl}\, k(r)\, \sin\theta_1\,, \quad T^r{}_{\theta_k\theta_l} = \epsilon_{kl}\, l(r)\, \sin\theta_1\,,$$
$$W_\lambda = (w_1(r), w_2(r), 0, 0)\,,$$

whereas the metric is in the standard spherically symmetric form:

$$ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2\right)\,.$$

Here, ϵ_{kl} is the Levi-Civita symbol in two dimensions.

Exact BH solution with dynamical torsion and nonmetricity

- The solution for the metric behaves as Reissner-Nordström

$$g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_{\rm s}^2 - 4e_1 \kappa_{\rm d,e}^2}{r^2}\,. \qquad (5)$$

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- Here κ_s is the spin charge (torsion) and $\kappa_{d,e}$ the dilation charge (nonmetricity).
- They involve a direct geometric effect on the motion of spinning (torsional part) and dilational (nonmetricity part) test bodies characterised by a general coupling to the torsion and nonmetricity.

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When these charges might be important?

Significant effects are contemplated only around **extreme gravitational systems**, such as **neutron stars** with intense magnetic fields and sufficiently oriented elementary spins or **black holes** endowed with spin and dilation charges.

Extension to axisymmetric space-times

- Metric, torsion and nonmetricity tensors in symmetric space-times:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q^\lambda{}_{\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,. \qquad (6)$$

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- Stationary and axisymmetric space-times

$$\#10 \to \#4 \begin{cases} ds^2 = \Psi_1(r,\vartheta)\, dt^2 - \frac{dr^2}{\Psi_2(r,\vartheta)} \\ -r^2\Psi_3(r,\vartheta)\left[d\vartheta^2 + \sin^2\vartheta(d\varphi - \Psi_4(r,\vartheta)dt)^2\right] \end{cases} ;$$

$$\#24 \left\{ T^\lambda{}_{\mu\nu} = T^\lambda{}_{\mu\nu}(r,\vartheta) \right. \tag{7}$$

$$\#4 \left\{ W_\mu = (W_t(r,\vartheta), W_r(r,\vartheta), W_\vartheta(r,\vartheta), W_\varphi(r,\vartheta))\,. \right. \tag{8}$$

Extension to axisymmetric space-times

- Rotating Kerr-Newman metric structure[3]:

$$ds^2 = \Psi(r,\vartheta)\, dt^2 - \frac{r^2 + a^2\cos^2\vartheta}{(r^2+a^2\cos^2\vartheta)\,\Psi(r,\vartheta) + a^2\sin^2\vartheta}\, dr^2$$
$$- \left(r^2 + a^2\cos^2\vartheta\right) d\vartheta^2 + 2a\left(1-\Psi(r,\vartheta)\right)\sin^2\vartheta\, dt d\varphi$$
$$- \sin^2\vartheta\left[r^2 + a^2 + a^2\left(1-\Psi(r,\vartheta)\right)\sin^2\vartheta\right] d\varphi^2\,, \quad (9)$$

$$\Psi(r,\vartheta) = 1 - \frac{\left[2mr + 4e_1(\kappa_{\mathrm{d,e}}^2 + \kappa_{\mathrm{d,m}}^2) - d_1\kappa_{\mathrm{s}}^2\right]}{r^2 + a^2\cos^2\vartheta}\,. \quad (10)$$

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- Field strength tensors:

$$\bar{R}_{[\mu\nu]} = \frac{1}{12}\varepsilon^{\lambda}{}_{\sigma\mu\nu}\nabla_\lambda \bar{S}^\sigma + \frac{1}{2}\nabla_\lambda \bar{t}^{\lambda}{}_{\mu\nu}\,; \quad \tilde{R}^{\lambda}{}_{\lambda\mu\nu} = 4\nabla_{[\nu}W_{\mu]}\,;$$
$$\bar{R}^{\lambda}{}_{[\mu\nu\rho]} = \frac{1}{6}\varepsilon^{\lambda}{}_{\sigma[\rho\nu}\nabla_{\mu]}\bar{S}^\sigma + \nabla_{[\mu}\bar{t}^{\lambda}{}_{\rho\nu]} + \frac{1}{4}\varepsilon^{\lambda}{}_{\omega\sigma[\rho}\mathring{t}_1^{\sigma}{}_{\mu\nu]}\bar{S}^\omega$$
$$- \frac{1}{18}\varepsilon_{\sigma\mu\nu\rho}\mathring{T}_1^{\lambda}\bar{S}^\sigma\,. \tag{11}$$

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Extension to axisymmetric space-times

- Nonmetricity sector:(no approx.)

$$
\begin{aligned}
w_1(r,\vartheta) &= \frac{\kappa_{\mathrm{d,e}} r - a\,\kappa_{\mathrm{d,m}}\cos\vartheta}{r^2 + a^2\cos^2\vartheta}\,, \quad w_3(r,\vartheta) = 0\,,\\
w_2(r,\vartheta) &= -\frac{\kappa_{\mathrm{d,e}} r}{(r^2 + a^2\cos^2\vartheta)\Psi(r,\vartheta) + a^2\sin^2\vartheta}\,,\\
w_4(r,\vartheta) &= \kappa_{\mathrm{d,m}}\left(\frac{r^2 + a^2}{r^2 + a^2\cos^2\vartheta}\cos\vartheta - \gamma\right) - a\,\frac{\kappa_{\mathrm{d,e}} r\sin^2\vartheta}{r^2 + a^2\cos^2\vartheta}\,. \qquad (12)
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- Torsion sector (decoupling limit between the spin and the orbital angular momentum $|a\kappa_{\rm s}| \ll 1$):

$$\bar{\mathcal{S}}^a = -\frac{\kappa_{\rm s}}{r}(1,1,0,0) + \mathcal{O}(a\kappa_{\rm s})\,, \quad (13)$$
$$\bar{\mathcal{T}}_2^{abc} = \frac{\kappa_{\rm s}}{3r}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \end{pmatrix} + \mathcal{O}(a\kappa_{\rm s})\,. \quad (14)$$

Gravitational spin-orbit interaction

- We found a solution in the decoupling limit $a\kappa_s \ll 1$, which ensures that the Maxwell equation and closure conditions are fulfilled by the field strength tensors of torsion

$$\nabla_\lambda \tilde{R}^\lambda{}_{[\rho\mu\nu]} \;=\; \nabla_\mu \tilde{R}^{[\mu\nu]} = 0\,, \quad \nabla_{[\sigma}\tilde{R}_{\lambda[\rho\mu\nu]]} = \nabla_{[\lambda}\tilde{R}_{[\mu\nu]]} = 0\,.$$

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- **Gravitational spin-orbit interaction:**

$$\mathcal{H}_{\rm LS} = \frac{1}{m_{\rm e}^2 r}\frac{\partial V}{\partial r}\mathbf{L}\cdot\mathbf{S} \approx \frac{d_1}{2r}\frac{\partial g_{tt}}{\partial r} a\kappa_{\rm s}\cos\vartheta \tag{15}$$

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- Our Kerr-Newman solution assumes the decoupling limit $a\kappa_s \ll 1$, but without assuming this, one might be able to find an exact solution with terms in the metric proportional to a and κ_s.
- This would trigger a new unexplored effect in physics, which would be a gravitational spin-orbit interaction. What physically can be the implications of this?