Generalised Nonminimally Gravity-matter Coupled Theory

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Outline

1. Teleparallel gravity and some modifications
2. Generalised nonminimally gravity-matter coupled theory
3. Conclusions

Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) ${e^m}_\mu$ which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters $\rightarrow$ space-time indices;
  Latin letters $\rightarrow$ tangent space indices; ${E_a}^\mu$ is the inverse of the tetrad.
- Tetrads satisfy the orthogonality condition: ${E_m}^\mu {e^n}_\mu = \delta^n_m$ and ${E_m}^\nu {e^m}_\mu = \delta^\nu_\mu$ and metric can be reconstructed via $g_{\mu\nu} = \eta_{ab} {e^a}_\mu {e^b}_\nu$

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Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

Weitzenböck connection

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^a{}_{\nu} = E_a{}^{\rho}(\partial_{\mu} e^a{}_{\nu} + \omega^a{}_{b\mu} e^b{}_{\nu}) \,.$$

- By using this connection, one can express the torsion tensor as follows

Torsion tensor

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu} \,.$$

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- The Weitzenböck connection $\tilde{\Gamma}^{\rho}{}_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^{\rho}{}_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^{\rho}{}_{\nu\mu} = \Gamma^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\mu\nu}\,,$$

where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

[illegible]

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$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} = 0.$$

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Ricci theorem

- In a gravitational theory which satisfies $\nabla_\lambda g_{\mu\nu} = 0$, one has that the curvature can be split as

Ricci theorem in TEGR

$$R^\lambda{}_{\mu\sigma\nu} = \bar{R}^\lambda{}_{\mu\sigma\nu} + \nabla_\sigma K_\nu{}^\lambda{}_\mu - \nabla_\nu K_\sigma{}^\lambda{}_\mu + K_\sigma{}^\lambda{}_\rho K_\nu{}^\rho{}_\mu - K_\sigma{}^\rho{}_\mu K_\nu{}^\lambda{}_\rho = 0\,,$$

where $\bar{R}^\lambda{}_{\mu\sigma\nu}$ is the curvature tensor computed with the Levi-Civita connection and $K^\rho{}_{\mu\nu} = \frac{1}{2}(T_\mu{}^\rho{}_\nu + T_\nu{}^\rho{}_\mu - T^\rho{}_{\mu\nu})$ is the contorsion tensor related to the torsion tensor $T_\mu{}^\rho{}_\nu$. ∇_μ is computed with respect to the Weitzenböck connection.

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- By contracting the curvature tensor with the metric $g^{\mu\nu}R^{\lambda}{}_{\mu\lambda\nu} \equiv R$ (Ricci scalar), one gets

Ricci theorem final in TEGR

$$R = \bar{R} + T - B = 0 \rightarrow \bar{R} = -T + B\,,$$

where $B = \frac{2}{e}\partial_\mu(eT^\mu)$ is a boundary term in the action (see later) and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$ is the scalar torsion.

- Here $e = \sqrt{-g} = \det(e^{\mu}_{a})$ and $T_\mu = T^{\lambda}{}_{\lambda\mu}$.

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Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\mathrm{TEGR}} = \int \left[T + 2\kappa^2 L_{\mathrm{m}}\right] e \, d^4x \, .$$

where $\kappa^2 = 8\pi G$, $e = \det(e^a_\mu) = \sqrt{-g}$ and L_{m} is any matter Lagrangian.

- T and the scalar curvature $\bar{R}$ differs by a boundary term B as $\bar{R} = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

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$f(T)$ gravity

- In analogy with $f(\bar{R})$ gravity, one can consider in the Teleparallel framework, the $f(T)$ gravity

$f(T)$ gravity action

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar T depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory**:

Not equivalency between $f(\bar{R})$ and $f(T)$

Field equations of $f(T) \neq$ Field equations of $f(\bar{R})$

- It is possible to find a function f which mimics ΛCDM without introducing any cosmological constant.

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$f(T,B)$ gravity

- It is possible to extend this theory by adding more invariants. One interesting theory is when one considers*

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$$S_{f(T,B)} = \int f(T,B)e\, d^4x \,.$$

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- If $f(T,B) = f(T)$, one gets $f(T)$ gravity
- New theories related to the boundary term such as $T + f(B)$ gravity.

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$f(T, B, L_{\rm m})$ gravity

- Inspired by theories with nonminimally gravity-matter couplings in the curvature approach and also from $f(T, B)$ gravity, let us now consider the following gravity model

Generalised nonminimally gravity-matter coupled theory

$$S_{f(T,B,L_{\rm m})} = \int e f(T, B, L_{\rm m})\, d^4x \,,$$

where the function f depends on the scalar curvature T, the boundary term B and the matter Lagrangian $L_{\rm m}$

- By varying this action with respect to the tetrads, one finds

$$2\nabla^{\beta}\nabla_{\lambda} f_B - 2\delta^{\beta}_{\lambda}\Box f_B - B f_B \delta^{\beta}_{\lambda} - 4\Big[(\partial_\mu f_T) + (\partial_\mu f_B)\Big] S_{\lambda}{}^{\mu\beta}$$

$$-4 f_T e^{a}_{\lambda}\Big(e^{-1}\partial_\mu(e S_a{}^{\mu\beta}) - T^{\sigma}{}_{\mu a} S_{\sigma}{}^{\beta\mu}\Big) + f\delta^{\beta}_{\lambda} - f_L L_{\rm m}\delta^{\beta}_{\lambda} = 2 f_L \mathcal{T}^{\beta}_{\lambda} \,.$$

where $f_T = \partial f/\partial T$, $f_B = \partial f/\partial B$ and $f_L = \partial f/\partial L_m$.

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where $f_T = \partial f/\partial T$, $f_B = \partial f/\partial B$ and $f_L = \partial f/\partial L_m$.

$f(T, B, L_{\rm m})$ gravity

- Inspired by theories with nonminimally gravity-matter couplings in the curvature approach and also from $f(T, B)$ gravity, let us now consider the following gravity model

Generalised nonminimally gravity-matter coupled theory

$$S_{f(T,B,L_{\rm m})} = \int e f(T, B, L_{\rm m})\, d^4x\,,$$

where the function f depends on the scalar curvature T, the boundary term B and the matter Lagrangian $L_{\rm m}$

- By varying this action with respect to the tetrads, one finds

$$2\nabla^{\beta}\nabla_{\lambda}f_B - 2\delta^{\beta}_{\lambda}\Box f_B - Bf_B\delta^{\beta}_{\lambda} - 4\Big[(\partial_{\mu}f_T) + (\partial_{\mu}f_B)\Big]S_{\lambda}{}^{\mu\beta}$$
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Conservation equation

- By taking covariant derivative in the field equations, and after some simplifications, we find that the conservation equation holds if one gets that the standard conservation equation for the energy-momentum tensor is satisfied if the function f satisfy the following form

$$\Big(2\mathcal{T}_{\mu\nu}+g_{\mu\nu}L_m\Big)\nabla^\mu f_L=-e^a_\mu g_{\beta\nu}\frac{\partial L_m}{\partial e^a_\beta}\nabla^\mu f_L=0\,.$$

- For flat FLRW cosmology and considering a perfect fluid, the above equation is satisfied for any f.
- Similarly as other modified theories like $f(\bar{R},L_m)$, one can chose $L_m=-2\rho$ in this case.

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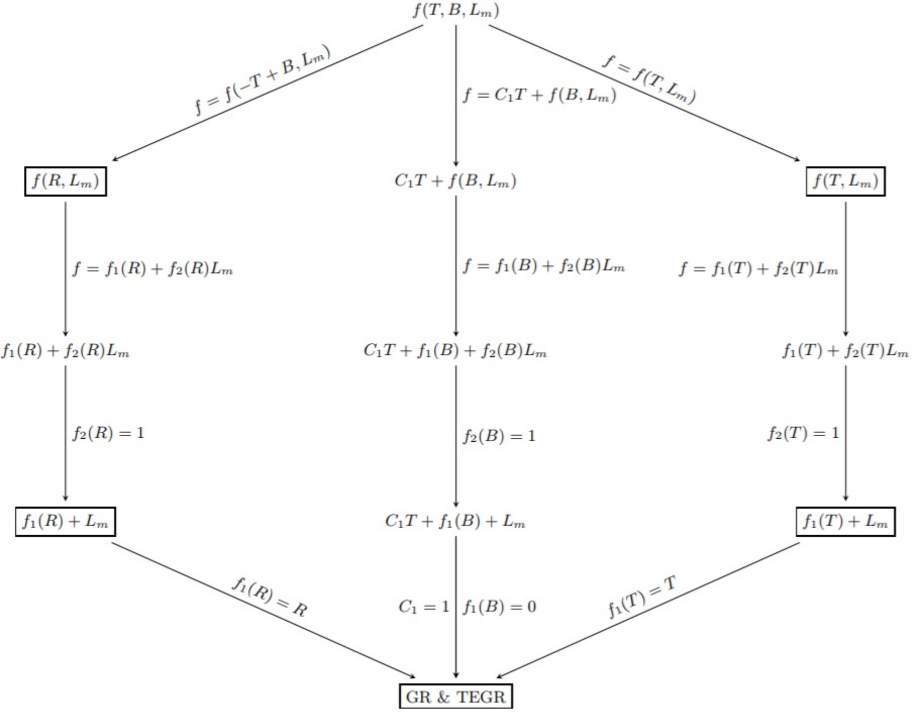
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- For flat FLRW cosmology $ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$ with $e^a_\beta = \text{diag}(1, a(t), a(t), a(t))$, one gets the following modified FLRW equations

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3H^2(3f_B + 2f_T) - 3H\dot{f}_B + 3f_B\dot{H} + \frac{1}{2}f &= 0\,, \\
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  where $H = \dot{a}/a$ is the Hubble parameter and dots represent derivation with respect to the cosmic time.
- It is easy to find that $T = -6H^2$ and $B = -6(\dot{H} + 3H^2)$.
- For $p = w\rho$, one can directly find that the energy density of the fluid behaves as $\rho(t) = \rho_0 a(t)^{-3(1+w)}$

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Briefly review about dynamical system in $f(T,B,L_m)$

- Let us now introduce the following dimensionless variables

$$x_1=\frac{\dot{T}f_{TB}}{2Hf_T}\,,x_2=\frac{\dot{B}f_{BB}}{2Hf_T}\,,x_3=\frac{\dot{B}f_{BT}}{2Hf_T}\,,x_4=\frac{\dot{T}f_{TT}}{2Hf_T}\,,y_1=\frac{Bf_B}{12H^2f_T}\,,$$
$$y_2=\frac{Tf_B}{12H^2f_T}=-\frac{f_B}{2f_T}\,,z=-\frac{f}{12f_TH^2}\,,\phi=\frac{3(w+1)\rho f_{BL}}{f_T}\,,$$
$$\alpha=\frac{3(w+1)\rho f_{TL}}{f_T}\,,\theta=\frac{(w+1)\rho f_L}{2f_TH^2}\,.$$

- Using these variables, the first Friedmann eq. becomes

$$x_1+x_2+y_1+z+\phi=1\,.$$

- The dynamical system becomes very complicated and the dimensionality of the model depends on the model chosen. For a general f, the DS is a 9 dimensional dynamical system.

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Case 1: $f(T, B, L_m) = f(-T + B, L_m) = f(\bar{R}, L_m)$ gravity

- The dynamical system becomes a 5-dimensional one and it was studied in R. P. L. Azevedo and J. Páramos, Phys. Rev. D **94** (2016) no.6, 064036.
- I checked that the full dynamical system reduces to the one found in that paper.
- de-Sitter solutions are possible even for non-vanishing energy density, with the coupling between curvature and matter driving the accelerated expansion of the Universe.
- If $f = M^4 \exp\left(\frac{\bar{R}}{6H_0^2} + \frac{L_m}{6H_0^2\kappa}\right)$ is exponential, they found one saddle point behaving as radiation-like and one unstable fixed point with a de Sitter solution.
- If $f = \left(\kappa M^2\right)^{-\varepsilon}\left(\kappa\bar{R} + L_m\right)^{(1+\varepsilon)}$, they found 4 critical points: only one can be understood as dark energy but it is unstable. The other points cannot be related to any known epochs of the history of the Universe.

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Case 2: $f(T,B,L_m) = f(T,L_m)$ gravity

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- If $f(T,L_m) = -\Lambda \exp\left[-\frac{1}{\Lambda}\Big(T + L_m\Big)\right]$, where Λ is a positive cosmological constant, the equations can be directly solved finding that the Universe is always expanding as a de-Sitter.
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Case 3: $f(T,B,L_m) = T + f(B,L_m)$ gravity

- The dynamical system becomes a 5-dimensional one.
- The model $f(T,B,L_m) = C_1 T + C_5 B^s + (C_4 + C_3 B^q) L_m$ was studied finding that depending on the parameters q and s one finds different cosmological descriptions.
- When $q = 1 - s$, the DS becomes a 3 dimensional one. Generically, for $s > 0$ (integer), the DS only has one critical point which can describe dark energy and can be stable.
- For $s = -2$, one gets two critical points and one can denotes a matter dominated era (being saddle) and a late-time accelerating era (being stable).

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Conclusions

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- In this work, we have found a general nonminimally gravity-matter theory which enclosed many theories coming from modified GR and modified TEGR.
- The DS of the model was studied and it was determined that these theories can be good candidates for describing dark energy. More further studies with other models can analyse this in more detail.
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