Different cosmological behaviours for different frames of $F(R)$ gravity

Sebastián Bahamonde

sebastian.beltran.14@ucl.ac.uk
Department of Mathematics, University College London.

Theoretical Physics seminar, Dipartimento di Fisica Università Federico II, Naples.

Work in collaboration with Sergei Odintsov, V.K. Oikonomou and Petr V. Tretyakov.

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Outline

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$F(R)$ gravity (Jordan frame)

- A well studied modification of GR is $F(R)$ gravity, which has the following action

$F(R)$ gravity action

$$S_{F(R)} = \frac{1}{2\kappa^2}\int F(R)\sqrt{-g}\,d^4x\,.$$

- Here, F is an arbitrary (sufficiently smooth) function of the Ricci scalar. G.R. is recovered if $F(R) = R$.
- Ricci scalar depends on second derivatives of the metric tensor $\rightarrow$ **Fourth order theory.**

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$F(R)$ gravity and a minimally coupled scalar tensor theory

- We will now conformally transform the later action in order to obtain the scalar-tensor Einstein frame counterpart theory.
- By introducing two auxiliary fields A and B, one can re-write the $F(R)$ action as

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\,\{B(R-A)+F(A)\}\ . \qquad (1)$$

- By varying with respect to $B \Longrightarrow A = R$.
- By varying with respect to $A \Longrightarrow B = F'(A)$.

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- Hence, the later action takes the following form

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left\{ F'(A)(R - A) + F(A) \right\} \ . \tag{2}$$

- Now, we will conformally transform the metric as

Conformal transformation

$$\tilde{g}_{\mu\nu} = \frac{1}{F'(A)} g_{\mu\nu} = e^{\sigma} g_{\mu\nu} \ , \tag{3}$$

which modifies the Ricci scalar $R \to \tilde{R}$. Here, $\sigma = -\ln F'(A)$ is a new scalar field defined in terms of A. Tildes refer to the Einstein frame.

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- By using this transformation, the action takes the following form,

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-\tilde{g}} \left\{ \tilde{R} - \frac{3}{2} \tilde{g}^{\mu\nu} \tilde{\partial}_\mu \sigma \tilde{\partial}_\nu \sigma - V(\sigma) \right\} , \tag{4}$$

where the energy potential was defined as
$V(\sigma) = \frac{A}{F'(A)} - \frac{F(A)}{F'(A)^2}$.

- **Notation reminder: Variables with tildes mean Einstein frame scalar-tensor theory and without tildes Jordan $F(R)$ frame.**

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- We can rescale the scalar field σ to put it into the canonical form by introducing $\phi = \sqrt{\frac{3}{2\kappa^2}}\sigma$, giving us the following scalar-tensor canonical scalar field action

Minimally coupled scalar-tensor theory (Einstein frame)

$$S = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa^2} - \frac{1}{2}\tilde{\partial}_\mu\phi\tilde{\partial}^\mu\phi - V(\phi) \right\} . \quad (5)$$

- Hence, the above action represents the resulting Einstein frame scalar tensor theory corresponding to the Jordan frame $F(R)$ gravity.
- Conversely, one can start with the above action (Einstein frame) and transform it to a Jordan frame $F(R)$ gravity via $\tilde{g}_{\mu\nu} = e^{-\sigma} g_{\mu\nu} = e^{\mp\sqrt{\frac{2}{3}}\kappa\phi} g_{\mu\nu}$

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- Since both theories are equivalent, depending on the problem studied, it might be convenient to do calculations in one frame than in the other.
- For example, in inflation is easier to work out within the context of scalar-tensor theory.
- We can ask the following question

Important question

These theories are equivalent but, are they actually physically equal?

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Flat Friedmann-Robertson-Walker Cosmology

In this talk, I will focus on a flat FRW background given by the following line-element

Flat FRW metric

$$ds_J^2 = -dt^2 + a(t)^2 \sum_{i=1,2,3} \left(dx^i\right)^2 \, , \tag{6}$$

where $a(t)$ is the scale factor of the universe in the Jordan $F(R)$ frame.
The homogeneous and isotropic principle implies the above space-time.

FRW Transformations between frames

- By performing the conformal transformation $g_{\mu\nu} \to e^{\pm\sqrt{\frac{2}{3}}\kappa\phi}\tilde{g}_{\mu\nu}$, the FRW metric will be

Line-elements after a conformal transformation

$$ds_J^2 = e^{\pm\sqrt{\frac{2}{3}}\kappa\phi}\left(-d\tilde{t}^2 + \tilde{a}(\tilde{t})^2 \sum_{i=1,2,3}\left(dx^i\right)^2\right), \quad (7)$$

FRW Transformations between Jordan and Einstein frames

$$dt = e^{\pm\frac{1}{2}\sqrt{\frac{2}{3}}\kappa\phi}d\tilde{t}\,, \quad (8)$$

$$a(t(\tilde{t})) = e^{\pm\frac{1}{2}\sqrt{\frac{2}{3}}\kappa\phi}\tilde{a}(\tilde{t})\,. \quad (9)$$

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FRW equations in both frames

FRW equations for scalar-tensor Einstein frame

$$3\tilde{H}^2 = \frac{1}{2}\dot{\phi}^2 + V\,, \tag{10}$$

$$3\tilde{H}^2 + 2\dot{\tilde{H}} = -\frac{1}{2}\dot{\phi}^2 + V\,. \tag{11}$$

FRW equations for $F(R)$ Jordan frame

$$0 = -\frac{F(R)}{2} + 3\left(H^2 + H'\right)F_R(R) - 18\left(4H^2H' + HH''\right)F_{RR}(R)\,, \tag{12}$$

$$0 = \frac{F(R)}{2} - \left(H' + 3H^2\right)F_R(R) + 6\left(8H^2H' + 4H'^2 + 6HH'' + H'''\right)F_{RR}(R) + 36\left(4HH' + H''\right)^2 F_{RRR}(R)\,. \tag{13}$$

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Do singularities change from one frame to another?

- Having in mind that one can transform from one frame to another by a conformal transformation, we will study what happens with cosmological time singularities when we conformally transform from one frame to another.
- The following question arise:

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Do singularities change from one frame to another?

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Classification of types of singularities

There are four types of finite time singularities which depends on $a(t), p_{\rm eff}, \rho_{\rm eff}$ and derivatives of the Hubble rate:

- Type I ("the Big Rip Singularity"): $a(t)$, $\rho_{\rm eff}$ and $p_{\rm eff}$ diverge as $t \to t_s$. (most severe)
- Type II ("Sudden Singularity"): $a(t)$ and $\rho_{\rm eff}$ remain bounded and $p_{\rm eff}$ diverges as $t \to t_s$.
- Type III: $a(t)$ remains bounded and $\rho_{\rm eff}$ and $p_{\rm eff}$ diverge as $t \to t_s$.
- Type IV: $a(t)$, $\rho_{\rm eff}$ and $p_{\rm eff}$ remain bounded but the second or higher derivatives of the Hubble rate diverge as $t \to t_s$ (least severe).

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Example I: Power-law cosmology from Einstein frame

- Let us consider a power-law cosmology $\tilde{a}(\tilde{t}) = \tilde{a}_0(\tilde{t}/\tilde{t}_0)^p$ where $\tilde{t}_0$ being some fiducial time and p a positive real free parameter. By using the FRW scalar-tensor eqs (Einstein frame) we can easily find that

$$\phi = \pm\sqrt{2p}\,\ln(\tilde{t}/\tilde{t}_0) \tag{14}$$

- In this case $\tilde{t}_1 = 0$ and $\tilde{t}_2 = \infty$ and in this model the Hubble rate $\tilde{H}$ diverges at $\tilde{t} = 0$, so we have a Type III singularity in the Einstein frame.

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- Let us now apply a conformal transformation to convert the theory to a Jordan frame. If we use the transformations found it before, we get

$$\frac{dt}{d\tilde{t}} = (\tilde{t}/\tilde{t}_0)^{\pm\sqrt{\frac{p}{3}}}, \rightarrow \quad t = \frac{3}{3 \pm \sqrt{3p}}\tilde{t}\left(\frac{\tilde{t}}{\tilde{t}_0}\right)^{\pm 2\sqrt{\frac{p}{3}}}, \tag{15}$$

$$\Longrightarrow a(t) \sim t^{\frac{\sqrt{3p}\pm 3p}{\sqrt{3p}\pm 3}}. \tag{16}$$

- For $-$: Type I finite time singularity at $t = 0$ if the power law parameter p lies in the range $1/3 \leq p < 3$.
- If $p = 1/3$, the Jordan frame metric becomes static and there is no longer a singularity.
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- The simplest singular cosmology is described by the following Hubble rate,

$$\tilde{H}(\tilde{t}) = f_0(\tilde{t} - \tilde{t}_s)^{\alpha}\,, \tag{17}$$

  with f_0 an arbitrary real and positive parameter and α a real number
- When $\alpha < -1$, the cosmology develops a Type I singularity.
- When $-1 < \alpha < 0$, the cosmology develops a Type III singularity.
- When $0 < \alpha < 1$, the cosmology develops a Type II singularity.
- When $\alpha > 1$, the cosmology develops a Type IV

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$$\tilde{H}(\tilde{t}) = f_0(\tilde{t} - \tilde{t}_s)^{\alpha}\,, \tag{17}$$

with f_0 an arbitrary real and positive parameter and α a real number

- When $\alpha < -1$, the cosmology develops a Type I singularity.
- When $-1 < \alpha < 0$, the cosmology develops a Type III singularity.
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Example II: A singular cosmological evolution from Einstein frame

One can follow the same procedure as before for this more general model and one can summarize the following:

Singularity in Einstein Frame	**Singularity in Jordan Frame**
Type I	Type I or no singularity
Type III	Type III
Type II	Type III
Type IV	Type IV or Type II

Table: Correspondence for finite time singularities in the Einstein and Jordan frames, for the cosmological evolution $\tilde{H}(\tilde{t}) = f_0(\tilde{t} - \tilde{t}_s)^{\alpha}$ in the Einstein frame.

Correspondence of $F(R)$ gravity singularities in different frames

We can conclude that:

First conclusion of this section

$F(R)$ gravity has an equivalent minimally coupled scalar tensor theory when one conformally transforms the metric as $g_{\mu\nu} = e^{\sigma}\tilde{g}_{\mu\nu}$.

Second conclusion of this section

Cosmological time singularities might change from one frame to another.

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Outline

1. Introduction
   - Jordan and Einstein frames
   - FRW cosmology in Jordan and Einstein frames
2. Correspondence of $F(R)$ Gravity Singularities in different frames
   - $F(R)$ gravity and scalar-tensor theory
3. Acceleration and Deceleration in different frames
   - Minimally curvature-coupled scalar-tensor theory
4. Conclusions

What happens with acceleration/deceleration?

- We showed that time singularities might change from one frame to another. So, now we will ask the following important question:

Question

Is it possible to have a situation where we have acceleration (deceleration) of the universe in one frame and deceleration (acceleration) of the universe in the other?

Main objectives of this section

We shall investigate under which circumstances, an accelerating evolution in one frame may be transformed to a decelerating evolution in the other frame

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How does acceleration/deceleration change from one frame to another?

- Reminder: the scale factors of the Jordan and Einstein frames are related as follows,

$$a(t(\tilde{t})) = e^{\frac{1}{2}\sqrt{\frac{2}{3}}\phi}\tilde{a}(\tilde{t}). \tag{18}$$

- The second derivative of the scale factor (acceleration) reads,

$$a'' = \left(\frac{1}{2}\sqrt{\frac{2}{3}}\ddot{\phi}\tilde{a} + \frac{1}{2}\sqrt{\frac{2}{3}}\dot{\phi}\dot{\tilde{a}} + \ddot{\tilde{a}}\right) e^{-\frac{1}{2}\sqrt{\frac{2}{3}}\phi}. \tag{19}$$

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Accelerating in the Einstein frame and decelerating in the Jordan frame

- We will study the case where we have acceleration in the Einstein frame ($\ddot{\tilde{a}} > 0$) and simultaneously deceleration in the Jordan frame ($a'' < 0$).
- In addition, the conditions $a' >$ and $\dot{\tilde{a}} > 0$ must hold true in order to have expansion in the both frames.
- In order for the above constraints to be satisfied, it suffices if the following conditions hold true,

$$A \equiv \frac{1}{2}\sqrt{\frac{2}{3}}\ddot{\phi}\tilde{a} + \frac{1}{2}\sqrt{\frac{2}{3}}\dot{\phi}\dot{\tilde{a}} + \ddot{\tilde{a}} > 0, \tag{20}$$

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- From the Einstein scalar-tensor eqs., it is easily to find that $\phi(\tilde{t}) = -\frac{2}{1+\alpha}\sqrt{-2f_0\alpha}\tilde{t}^{\frac{\alpha+1}{2}}$. Hence, $\alpha < -1$ and $f_0 > 0$.
- Therefore, it is easy to find that

$$a' = a f_0 \tilde{t}^{\alpha}\left(1 + \sqrt{\frac{-\alpha}{3f_0}}\tilde{t}^{\frac{-\alpha-1}{2}}\right), \tag{22}$$

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- For the acceleration, we have

$$\ddot{\tilde{a}} = \tilde{a} f_0 \tilde{t}^{\alpha-1}\left(\alpha + f_0 \tilde{t}^{\alpha+1}\right) . \quad (24)$$

- This function may have different sign for the parameters chosen.
- According to our previous considerations, we are interested in the case that $\ddot{a} < 0$ and $A > 0$ or equivalently,

$$\frac{\alpha-1}{2}\sqrt{\frac{-f_0\alpha}{3}} + \sqrt{\frac{-f_0\alpha}{3}} f_0 \tilde{t}^{\alpha+1} + f_0^2 \tilde{t}^{\frac{3\alpha+3}{2}} + f_0 \alpha \tilde{t}^{\frac{\alpha+1}{2}} > 0 , \quad (25)$$

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- It is expected that for some specific times, these inequalities will hold.
- Let us suppose that the first inequality is true for t_*, so we may put,

$$\tilde{t}_*^{\alpha+1} = m\frac{-\alpha}{f_0}, \tag{27}$$

where $0 < m < 1$ is some numerical parameter.

- Substituting this expression in the second inequality, we find,

$$B \equiv \frac{\alpha - 1}{2\sqrt{3}} - \frac{\alpha m}{\sqrt{3}} + m^{\frac{3}{2}}|\alpha| + m^{\frac{1}{2}}\alpha > 0. \tag{28}$$

- If the sum of the first two terms are positive, the above inequality will hold.

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- A detailed analysis of this inequality, imposes an additional restriction for the parameter, namely $\frac{1}{2} < m < 1$.
- By taking into account the negative values of α, the later inequality becomes

$$|\alpha| > \frac{1}{2\sqrt{3}m^{\frac{3}{2}} + 2m - 2\sqrt{3}m^{\frac{1}{2}} - 1}. \quad (29)$$

- The expression in the denominator is monotonically increasing function of m in the range $\frac{1}{2} < m < 1$, which crosses zero near the point $m \approx 0.8042$.
- All interesting values $0.8042 \lesssim m < 1$, for example $m = 0.9$ gives us $B \simeq 0.119 > 0$.

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- Therefore, all the inequalities can be true for some specific values of the parameters.

Result of the calculations:

We explicitly demonstrated for a specific model that for some parameters, simultaneously we might have an accelerating behaviour of the universe in the Einstein frame and a deceleration behaviour in the Jordan frame.

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Note on non-minimally scalar tensor theory

What happens with a non-minimally scalar tensor theory?

By using conformal transformations, it is also possible to relate a non-minimally coupled tensor theory (a Jordan frame) with a specific minimally coupled tensor theory (an Einstein frame). Doing a similar analysis as before, It can be also shown that it is possible to have the situation where we have acceleration in one frame and deceleration in the other.

Outline

Conclusions

- We presented how $F(R)$ gravity (Jordan frames) are equivalent to a specific minimally coupled scalar-tensor theory (Einstein frame) when we conformally transform the metric as $\tilde{g}_{\mu\nu} = e^{-\sigma} g_{\mu\nu}$.
- We presented how these two frames are mathematically related.
- We studied that cosmological time singularities might change from one frame to another.

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- We also showed that this behaviour is even more severe: There are situations in which we can have acceleration of the universe in one frame whereas we have deceleration of the universe in the other.
- With some specific models we explicitly showed this difference and how this scenario can be achieved for the Jordan frame and the Einstein frame.
- Therefore, doing a general analysis, we showed that even though these theories are mathematically equivalent, the physical interpretation of them might be different.

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