# Solar System Tests in Modified Teleparallel gravity

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# Outline

# Teleparallel equivalent of General Relativity action

- The TEGR action is formulated based on the torsion scalar $T$, namely

$$S_{\rm TEGR} = \int \left[T + 2\kappa^2 L_{\rm m}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e^a_\mu) = \sqrt{-g}$ and $L_{\rm m}$ is any matter Lagrangian.

- $T$ and the scalar curvature $\mathring{R}$ differs by a boundary term $B$ as $\mathring{R} = -T + B$ so:

**Equivalence between field equations**

The teleparallel field equations are equivalent to the Einstein field equations.

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# Two different ways of understanding gravity

## Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them.

## Validity of TEGR

VERY IMPORTANT POINT: All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

## What happens if we modify TEGR?

If we modify the TEGR action, a priori there is no equivalence between modified theories from GR and modified Teleparallel theories.

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# $f(T)$ gravity

- In analogy with $f(\mathring{R})$ gravity, one can consider

**$f(T)$ gravity action**

$$S_{f(T)} = \int f(T) e \, d^4x \, .$$

- The torsion scalar $T$ depends on the first derivatives of the tetrads $\rightarrow$ **Second order theory**:

**Not equivalency between $f(\mathring{R})$ and $f(T)$**

Field equations of $f(T) \neq$ Field equations of $f(\mathring{R})$

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- If $f(T,B)=f(-T+B)=f(\mathring{R})$, one finds the $f(\mathring{R})$ theory in the context of TEGR.
- If $f(T,B)=f(T)$, one gets $f(T)$ gravity
- Other theories related to the boundary term such as $T+f(B)$ gravity.

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# Spherical symmetry in $f(T, B)$ gravity

- Let us assume a spherically symmetric spacetime whose metric is

$$ds^2 = \mathcal{A}(r)dt^2 - \mathcal{B}(r)dr^2 - r^2 d\Omega^2 \,,$$

where $\mathcal{A}(r)$ and $\mathcal{B}(r)$ are positive functions, which is reproduced by the off-diagonal tetrad

$$e^a{}_\mu = \begin{pmatrix} \sqrt{\mathcal{A}} & 0 & 0 & 0 \\ 0 & \sqrt{\mathcal{B}}\sin(\theta)\cos(\phi) & r\cos(\theta)\cos(\phi) & -r\sin(\theta)\sin(\phi) \\ 0 & \sqrt{\mathcal{B}}\sin(\theta)\sin(\phi) & r\cos(\theta)\sin(\phi) & r\sin(\theta)\cos(\phi) \\ 0 & \sqrt{\mathcal{B}}\cos(\theta) & -r\sin(\theta) & 0 \end{pmatrix} .$$

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# Perturbations around Schwarzschild

- Finding new exact solutions is difficult in $f(T, B)$, there have been some attempts but none of them have found physically interesting solutions so far.†
- Let us then assume that the background is described by the Schwarzschild geometry and the perturbed coefficients are first order corrections to this spacetime, namely,

$$\mathcal{A}(r) = 1 - \frac{2M}{r} + \epsilon\, a(r)\,,$$
$$\mathcal{B}(r) = \left(1 - \frac{2M}{r}\right)^{-1} + \epsilon\, b(r)\,.$$

Here $\epsilon \ll 1$ is a small tracking parameter.

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# Model studied

- To include different power-law forms of the Lagrangian, let us assume the following combination of power-law terms

$$f(T,B) = T + \frac{1}{2}\epsilon\left(\alpha T^q + \beta B^m + \gamma B^s T^w + \zeta(\xi T + \chi B)^u\right) \,,$$

where $\alpha, \beta, \gamma, \zeta, q, m, s, w$ and $u$ are constants.

- The case $\beta = \gamma = \zeta = 0$ (power-law $f(T)$) was studied before [‡].

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# Perturbed solutions

- The Cases 1a-1d have the following solutions:

$$\begin{aligned}\mathcal{A}(r) &= \mu^2-\frac{1}{(\mu^2-1)^2r^2}\Big[3\beta\mu^7-\frac{1}{2}(\alpha+13\beta)\mu^6-4\beta\mu^5+\frac{1}{2}(15\alpha+43\beta)\mu^4\\ &-\frac{2}{3}(32\alpha+35\beta)\mu^3+\frac{1}{2}(33\alpha+13\beta)\mu^2+4\beta\mu-\frac{1}{6}(13\alpha+\beta)-\frac{\beta}{\mu}\\ &-2(\alpha+\beta)\left(1-3\mu^2\right)\log\mu\Big]+\epsilon\,\tilde{a}_\gamma(r)\,,\\ \mathcal{B}(r) &= \mu^{-2}+\frac{\epsilon}{r^2(\mu^2-1)}\Big[\frac{1}{2}(25\alpha+37\beta)-4(\alpha+2\beta)\mu-\frac{2(16\alpha+13\beta)}{3\mu}\\ &-\frac{2(\alpha+3\beta)}{\mu^2}+\frac{4(\alpha+\beta)}{\mu^3}+\frac{\alpha-11\beta}{6\mu^4}+\frac{2\beta}{\mu^5}+\frac{2(\alpha+\beta)\log(\mu)}{\mu^4}\Big]+\epsilon\,\tilde{b}_\gamma(r)\,,\end{aligned}$$

where $\mu^2=1-2M/r$.

- $\tilde{a}_\gamma(r)$ and $\tilde{b}_\gamma(r)$ depend on the model. All the solutions are asymptotically flat.
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# Photon sphere and Shadow of the black hole

- the shadow of the black hole predicted by power-law $f(T,B)$ will be enlarged or reduced for each solution.
- The term $\alpha T^2$: the shadow of the black holes will be enlarged (reduced) if $\alpha > 0$ ($\alpha < 0$).
- The same happens for the $\gamma B^2$ and $\gamma BT$ (Case 1b), i.e., when $\beta, \gamma > 0$ ($\beta, \gamma < 0$), the shadow will be bigger.
- On the contrary for the contributions $\gamma BT^2$ (Case 1c) and $\gamma B^2 T$ (Case 1d), one needs $\gamma < 0$ ($\gamma > 0$) for a larger (smaller) shadow.

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# Perihelion shift

- For all the perturbed solutions found in the previous section, we find that the perihelion shift is given by $(q = M/r_c)$

$$\begin{aligned}\Delta\phi &= \Delta\phi_{\mathrm{GR}} + \epsilon\,\Delta\phi_{\epsilon} \\ &\approx 6\pi q + 27\pi q^2 + 135\pi q^3 + \mathcal{O}(q^4) + \\ &\quad \epsilon\,\pi\Big(\frac{12\beta q}{r_c^2} + \frac{8q^2(\alpha+10\beta)}{r_c^2} + \frac{q^3(194\alpha+1139\beta)}{2r_c^2} + \gamma\Delta\phi_\gamma\Big)\,,\end{aligned}$$

where $\Delta\phi_\gamma$ is:

- **Case 1a:** $\gamma = 0$ (If $\alpha, \beta > 0$, $\Delta\phi$ is bigger)
- **Case 1b:** $\Delta\phi_\gamma = \frac{44q^2}{r_c^2} + \frac{6q}{r_c^2} + \frac{1333q^3}{4r_c^2}$ (If $\gamma > 0$, $\Delta\phi$ is bigger)
- **Case 1c:** $\Delta\phi_\gamma = -\frac{112q^3}{r_c^4}$ (If $\gamma > 0$, $\Delta\phi$ is smaller)
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- **Case 1b:** $\Delta\phi_\gamma = \frac{44q^2}{r_c^2} + \frac{6q}{r_c^2} + \frac{1333q^3}{4r_c^2}$ (If $\gamma > 0$, $\Delta\phi$ is bigger)
- **Case 1c:** $\Delta\phi_\gamma = -\frac{112q^3}{r_c^4}$ (If $\gamma > 0$, $\Delta\phi$ is smaller)
- **Case 1d:** $\Delta\phi_\gamma = -\frac{56q^3}{r_c^4}$ (If $\gamma > 0$, $\Delta\phi$ is smaller)

# Deflection of light

- Following a similar computation as before, we find that the deflection angle for all the models can be expressed as

$$\vartheta = \vartheta_{\rm GR} + \epsilon\,\vartheta_\epsilon \approx \frac{4M}{r_0} + \frac{M^2}{r_0^2}\left(\frac{15\pi}{4} - 4\right) + \frac{M^3}{r_0^3}\Big(\frac{244-45\pi}{6}\Big) + \epsilon\left[\frac{4M^3}{15r_0^5}(16\alpha+\beta) + \gamma\vartheta_\gamma\right],$$

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# Shapiro Delay

- The Shapiro effect represents the time correction for the round trip of a radar signal that passes near a massive object in the presence of gravity.
- The time retardation for all the solutions can be written as,

$$\begin{aligned} t_{\rm S}(r,r_0) &= t_{\rm S,GR}(r,r_0)+\epsilon\, t_{\rm S,\epsilon}(r,r_0) \\ &\approx M\Big[1+2\log\Big(\frac{2r}{r_0}\Big)-\frac{r_0}{r}\Big]+\epsilon\Big[\frac{8\alpha M^3}{3r_0^4}-\frac{\beta M^2}{r r_0^2}+\gamma\, t_{\rm S,\gamma}\Big], \end{aligned}$$

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# Constraining the models

- For example, for the perihelion shift of Mercury, for GR the computation gives

$$\Delta\phi_{\rm GR,Mercury} \approx 0.1033\,\text{"/cycles} \approx 42,84\,\text{"/cen}\,.$$

- The observed value is $42,98 \pm 0.040\,$"/cen, so $\Delta\phi_\epsilon$ must lie between the error bars:

$$\Delta\phi_{\epsilon,\max} \approx 0.18\,\text{"/cen}\,.$$

- For example, for the Case 1a, one gets that the maximum value that the constants could be are

$$\Big|\alpha + 5.65\times 10^7\beta\Big|_{\max} \approx 3.65\times 10^{20}\,{\rm km}^2\,.$$

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# Conclusions

- We have found 6 new spherically symmetric solutions using perturbation methods around a Schwarzschild background in $f(T, B)$ gravity.
- We computed 6 different Solar System tests for all the models: photon sphere, perihelion shift, deflection of light, Shapiro delay, Cassini experiment and Grav redshift.
- In all the Solar System tests, the constraints are obtained by comparing the extra leading order terms produced by the particular phenomena against the analog GR term.
- The contributions $\gamma BT^2$ and $\gamma B^2T$ are much smaller than $\alpha T^2, \beta B^2$ and $\gamma BT$.

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