Observational constraints in metric-affine gravity

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Jointly with Jorge Gigante Valcarcel.

Outline

MAG models with dynamical torsion and nonmetricity

- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry[1]:

$$S = \int d^4x\sqrt{-g}\Big\{\mathcal{L}_{\rm m} + \frac{1}{64\pi}\Big[-\frac{4c^4}{G}R - 6d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\lambda[\rho\mu\nu]}$$
$$- 9d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\mu[\lambda\nu\rho]} + 8\,d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{[\mu\nu]} + \frac{1}{8}\left(32e_1 + 8e_2 + 17d_1\right)\tilde{R}^{\lambda}{}_{\lambda\mu\nu}\tilde{R}^{\rho}{}_{\rho}{}^{\mu\nu}$$
$$- 7d_1\tilde{R}_{[\mu\nu]}\tilde{R}^{\lambda}{}_{\lambda}{}^{\mu\nu} + 3\left(1-2a_2\right)T_{[\lambda\mu\nu]}T^{[\lambda\mu\nu]}\Big]\Big\}\,.$$

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- By varying this action w/r to the vierberin and the connection we arrive at two set of equations of the form

$$X1_{\mu}{}^{\nu}=16\pi\theta_{\mu}{}^{\nu}\,,$$
$$X2^{\lambda\mu\nu}=16\pi\triangle^{\lambda\mu\nu}\,,$$

where $\theta_{\mu}{}^{\nu}$ and $\triangle^{\lambda\mu\nu}$ are the canonical energy-momentum and hypermomentum density tensors of matter. We will concentrate on vacuum.

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- For more details about the theoretical formulation, see

Spherical symmetry

- Metric, torsion and nonmetricity in spherically symmetric space-times ($\#2 + \#8 + \#2 = \#12$):

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi W_\mu = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda\rho\mu\nu} = 0$$

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- By solving these equations we find that torsion and nonmetricity behave as

$$\begin{aligned} T^t{}_{tr} &= a(r)\,, \quad T^r{}_{tr} = b(r)\,, \quad T^{\theta_k}{}_{t\theta_k} = f(r)\,, \quad T^{\theta_k}{}_{r\theta_k} = g(r) \\ T^{\theta_k}{}_{t\theta_l} &= e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, d(r)\,, \quad T^{\theta_k}{}_{r\theta_l} = e^{a\theta_k}\, e^b{}_{\theta_l}\, \epsilon_{ab}\, h(r)\,, \\ T^t{}_{\theta_k\theta_l} &= \epsilon_{kl}\, k(r)\, \sin\theta_1\,, \quad T^r{}_{\theta_k\theta_l} = \epsilon_{kl}\, l(r)\, \sin\theta_1\,, \\ W_\lambda &= (w_1(r), w_2(r), 0, 0)\,, \end{aligned}$$

whereas the metric is in the standard spherically symmetric form:

$$ds^2 = c^2\Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\theta_1^2 + \sin\theta_1^2 d\theta_2^2\right)\,.$$

Here, ϵ_{kl} is the Levi-Civita symbol in two dimensions.

Spherical symmetry - Solving the field equations

The field eqs are very involved. To solve them we use the following strategy:

1. **Imposing regularity:** In general, the solutions can have a singular behaviour. To ensure regularity, one can analyse the torsion/non-metricity tensors referred to the rotated basis $\vartheta^a = \Lambda^a{}_b e^b$.

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   Regularity restricts the initial arbitrariness of the torsion components and the Weyl vector by imposing the relations

$$b(r) = c\,a(r)\,\sqrt{\Psi_1(r)\Psi_2(r)}\,, \qquad f(r) = -\,c\,g(r)\,\sqrt{\Psi_1(r)\Psi_2(r)}\,,$$
$$d(r) = -\,c\,h(r)\,\sqrt{\Psi_1(r)\Psi_2(r)}\,, \quad l(r) = c\,k(r)\,\sqrt{\Psi_1(r)\Psi_2(r)}\,,$$
$$w_1(r) = -\,c\,w_2(r)\sqrt{\Psi_1(r)\Psi_2(r)}\,.$$

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2. **Solve the weak field limit:** The weak field limit of the field equations become

$$\nabla_\rho \nabla_\lambda T^{\lambda\rho}{}_{\mu} + \nabla_\rho \nabla^\rho T^{\lambda}{}_{\mu\lambda} - \nabla_\rho \nabla_\mu T^{\lambda\rho}{}_{\lambda} = 0\,,$$
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These equations can be solved, yielding

$$w_1(r) = -\,c\,\kappa_d \int \sqrt{\frac{\Psi_1(r)}{\Psi_2(r)}}\,\frac{dr}{r^2}\,,$$

$$b(r) = r f'(r) + f(r) + \frac{c\,\kappa_d}{2r}\sqrt{\frac{\Psi_1(r)}{\Psi_2(r)}}\,,$$

where κ_d is an integration constant which represents the dilaton charge.

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3. **Use the field equations:** After the first two steps, the system is reduced and can be solved. From them, one finds that $e_2 = -d_1/2$ and $\Psi_1(r) = \Psi_2(r)$.

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   One finds that the unique solution behaves similarly as the Reissner-Nordström:

$$\Psi(r) = 1 - \frac{2GM}{c^2 r} + \frac{G\left(d_1\kappa_s^2 - 4e_1\kappa_d^2\right)}{c^4 r^2}\,.$$

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The torsion and nonmetricity tensors are independent. To see their explicit form, check[2]

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Nature of the black hole solution

- The nature of the horizons depends on the difference $d_1\kappa_s^2 - 4e_1\kappa_d^2$. Thus, a positive difference of this quantity would present two horizons determined from the roots

$$r_\pm = \frac{G}{c^2}\left(M \pm \Delta_1\right)\,, \quad \Delta_1^2 = M^2 - \frac{1}{G}\left(d_1\kappa_s^2 - 4e_1\kappa_d^2\right)\,,$$

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- The balance between κ_d and κ_s **is not restricted** to any special constraint and therefore any of these situations may occur in the presence of torsion and nonmetricity.
- In particular, we can have a RN type of solution with a unique event horizon and a different sign than the one provided by the Einstein-Maxwell model.

Particle motion in MAG

- The equations of motion of test bodies with microstructure coupled to the torsion and nonmetricity tensors become[3]

$$\dot{p}^{\mu} + \Gamma^{\mu}{}_{\lambda\rho}\, p^{\lambda} u^{\rho} + N_{[\lambda\rho]}{}^{\mu} p^{\rho} u^{\lambda} + \tilde{R}_{\lambda\rho\sigma}{}^{\mu} \triangle^{\rho\lambda} u^{\sigma} = 0\,.$$

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- For the particle motion of bosons, the above equation is just the standard geodesic equation. We will concentrate on effects on such particles.

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- For the particle motion of bosons, the above equation is just the standard geodesic equation. We will concentrate on effects on such particles.
- Using the standard geodesic approach, we find

$$\frac{1}{2}\dot{r}^2 + V(r) = 0\,, \quad V(r) = -\frac{1}{2}c^2 E^2 + \frac{1}{2}\Psi(r)\left(\frac{J^2}{r^2} + \sigma c^2\right)\,,$$

where E an J are the conserved charges and $\sigma = 0(\sigma = 1)$ represents massless(massive) particles.

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Photon sphere and perihelion shift

- **Photon sphere:** Region of space where gravity is so strong that photons are forced to travel in orbits. They are found by setting $V'(r) = V(r) = \sigma = 0$, giving us:

$$r_1 = \frac{G}{2c^2}\left(3M + \Delta_2\right) , \quad J_{1,\pm} = \pm\frac{GE(\Delta_2 + 3M)^2}{\sqrt{2}c\sqrt{\Delta_2^2 + 3M^2 + 4\Delta_2 M}} ,$$

$$r_2 = \frac{G}{2c^2}\left(3M - \Delta_2\right) , \quad J_{2,\pm} = \pm\frac{GE(\Delta_2 - 3M)^2}{\sqrt{2}c\sqrt{\Delta_2^2 + 3M^2 - 4\Delta_2 M}} ,$$

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$\Delta_2^2 := M^2 + 8\Delta_1^2 = 9M^2 - \frac{8}{G}\left(d_1\kappa_s^2 - 4e_1\kappa_d^2\right)$.

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- The first pair $(r_1, J_{1,\pm})$ describes a unique photon sphere that lies outside the event horizons, with the corrections related to κ_s, κ_d affecting its location with respect to the Schwarzschild solution of GR.

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- **Perihelion shift:** We consider a massive body with $\sigma = 1$ and a perturbation around its closed orbit r_c ($\dot{r}_c = V(r_c) = V'(r_c) = 0$):

$$\Delta\phi = 2\pi\left[\frac{3GM}{c^2 r_c} + \frac{27G^2M^2}{2c^4r_c^2} + \frac{135G^3M^3}{2c^6r_c^3} + \frac{2835G^4M^4}{8c^8r_c^4} - d_1\kappa_s^2\left(\frac{1}{2c^2Mr_c} + \frac{6G}{c^4r_c^2}\right) + e_1\kappa_d^2\left(\frac{2}{Mc^2r_c} + \frac{24G}{c^4r_c^2}\right)\right].$$

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- One could expect that these contributions coming from metric-affine geometry will be only sourced in a strong gravitational regime, e.g., Sgr A* and S2 stars.

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- One could expect that these contributions coming from metric-affine geometry will be only sourced in a strong gravitational regime, e.g., Sgr A* and S2 stars.
- Since $\Delta\phi_{\mathrm{S}_2}^{(\mathrm{GR})} \approx 48.550\,[''/\mathrm{year}]$ and $\Delta\phi_{\mathrm{S}_2}^{(\mathrm{obs})} = 48.506 f_{\mathrm{SP}}\,[''/\mathrm{year}]$[4] we find the constrain

$$4e_1\kappa_d^2 - 5.711\cdot 10^{63}\ [J\cdot m] \leq d_1\kappa_s^2 \leq 4e_1\kappa_d^2 + 2.894\cdot 10^{63}\ [J\cdot m]\ .$$

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Gravitational redshift

- Another gravitational effect that can be used to constrain the new effects arising from our model is the gravitational redshift, that for our solution we get

$$z = \frac{GM}{c^2 R} + \frac{3G^2M^2}{2c^4R^2} + \frac{5G^3M^3}{2c^6R^3} + \frac{35G^4M^4}{8c^8R^4} + \frac{G\left(4e_1\kappa_d^2 - d_1\kappa_s^2\right)}{2c^4R^2}\,.$$

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- Current measurements for the masses and gravitational redshifts of isolated neutron stars do not provide independent quantities.
- We focused on the Sirius B white dwarf, whose Doppler shift velocity $v = c\,z$ with $v_{\rm obs,SiriusB} = (80.65 \pm 0.77)\,[\frac{\rm km}{s}]$ and $v_{\rm GR,SiriusB} = 80.0464\,[\frac{\rm km}{s}]$, which gives us

$$4e_1\kappa_d^2 - 2.931\cdot 10^{43}\ [J\cdot m] \leq d_1\kappa_s^2 \leq 4e_1\kappa_d^2 + 1.016\cdot 10^{43}\ [J\cdot m]\,.$$

Combined observations

- Let us now consider the case where the effect of torsion dominates over the contribution of nonmetricity.

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- To the best of our knowledge, this bound provides the first observational comparison between the spin charges of a supermassive black hole and a degenerate star.

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- Further, we provided a bound of the spin charge of SgrA* related to Sirius B.
- In our paper we also calculated two other observables (Shapiro delay and deflection of light), but there are not reliable measurements that can be used for our theory to constrain the graviational effects of κ_s and κ_d.