Generalised teleparallel quintom dark energy non-minimally coupled with the scalar torsion and a boundary term

Sebastián Bahamonde

PhD student at Department of Mathematics, University College London.

TeleGrav, University of Tartu

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Based on S. Bahamonde, M. Marciu and P. Rudra, JCAP **1804** (2018) no.04, 056.

Outline

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What are dynamical systems?

- It is a framework which can be used to study something as simple as a single pendulum to something as complicated as the dynamics of the universe which possess
  - a space (*state space* or *phase space*), and
  - a mathematical rule describing the evolution of points in that space.
- If $x = (x_1, x_2, \ldots, x_n) \in X$ is an element of the state space $X \subseteq \mathbb{R}^n$. A dynamical system is generally written in the form

$$\dot{x} = \mathbf{f}(x) \, ,$$

where the function $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \cdots, f_n(\mathbf{x}))$ and $t \in \mathbb{R}$ is a suitable parameter.

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Critical points and Linear stability theory

Definition (*Critical point* or *fixed point* or *equilibrium point*)

The autonomous equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ is said to have a critical (or fixed or equilibrium) point at $\mathbf{x} = \mathbf{x}_0$ if and only if $\mathbf{f}(\mathbf{x}_0) = 0$.

- Linear stability: We can linearise the system around its critical point, then using Taylor expansions one can define the *stability matrix*:

$$J = \frac{\partial f_i}{\partial x_j} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix},$$

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How to determine if the critical points are stable?

- If all eigenvalues of the Jacobian matrix have positive real parts, trajectories are repelled from the fixed point and we speak in this case of an *unstable point*
- If all eigenvalues have negative real parts, the point would attract all nearby trajectories and is regarded as *stable* and sometimes called *attractor* or *attracting node*.
- If at least two eigenvalues have real parts with opposite signs, then the corresponding fixed point is called a *saddle* point, which attracts trajectories in some directions but repels them along others.
- Linear stability theory fails for critical points where the Jacobian matrix $J(\mathrm{x}_0)$ have zero real part. Other methods like Lyapnuov or centre manifold theory.

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Example for flat ΛCDM cosmology

- Let us consider flat FLRW cosmology with a cosmological constant Λ and two fluids representing radiation and matter:

$$3H^2 = \kappa^2 \rho_{\rm m} + \kappa^2 \rho_{\rm r} + \Lambda\,,$$
$$2\dot{H} + 3H^2 = -\frac{\kappa^2}{3}\rho_{\rm r} + \Lambda\,,$$

The system above suggests the introduction of the dimensionless variables

$$x = \Omega_{\rm m} = \frac{\kappa^2 \rho_{\rm m}}{3H^2}, \quad y = \Omega_{\rm r} = \frac{\kappa^2 \rho_{\rm r}}{3H^2} \quad \text{and} \quad \Omega_\Lambda = \frac{\kappa^2 \rho_\Lambda}{3H^2}\,.$$

Example for flat ΛCDM cosmology

- We can now rewrite the first Friedmann equation as

$$1 = x + y + \Omega_\Lambda\,.$$

- To obtain the dynamical system, we differentiate the variable x and y with respect to $\eta = \log a$, which represents our dimensionless time variable. We obtain

$$x' = \frac{dx}{d\eta} = x\,(3x + 4y - 3)$$
$$y' = \frac{dy}{d\eta} = y\,(3x + 4y - 4)\ .$$

and then $w_{\rm eff} \equiv \frac{p_{\rm tot}}{\rho_{\rm tot}} = -1 + x + \frac{4}{3}y$.

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Example for flat ΛCDM cosmology

Point	x	y	$w_{\rm eff}$	Eigenvalues	Stability
O	0	0	-1	$\{-4,-3\}$	Stable point
R	0	1	1/3	$\{1,4\}$	Unstable point
M	1	0	0	$\{-1,3\}$	Saddle point

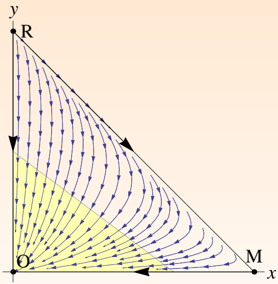

Figure: Phase space portrait of the dynamical system. The yellow/shaded area denotes the region of the phase space where the universe is accelerating.

Outline

Nonminimally coupled scalar fields with the scalar curvature

- One modification that it is important for our presentation is considering scalar fields nonminimally coupled with gravity. For example, a coupling with the scalar field ϕ to the scalar curvature R as follows (scalar-tensor theories)

$$S = \int \left[F(\phi)R + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi) + L_{\rm m} \right] \sqrt{-g}\, d^4x\,.$$

  Here $V(\phi)$ is an energy potential energy.
- This action has the following theories:
  - $F = 1$: minimally coupled (GR+ scalar field).
  - $F = \phi$: Brans-Dicke theory (first scalar modified gravity).
  - $F = 1 + \phi^2$: nonminimally coupled with interesting applications in cosmology.

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Nonminimally coupled scalar fields to the scalar torsion

An alternative approach has been to consider a scalar field nonminimally coupled to torsion. The following action is considered

$$S = \int \left[F(\phi) T + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L_{\rm m} \right] e \, d^4 x \,.$$

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With the aim of unifying both of the previous considered approaches, we proposed a more general action given by (S. Bahamonde and M. Wright Phys. Rev. D **92** (2015) no.8, 084034); M. Zubair, S. Bahamonde and M. Jamil, Eur. Phys. J. C **77** (2017) no.7, 472)

$$S = \int \left[F(\phi)T + G(\phi)B + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + L_{\rm m} \right] e\, d^4x.$$

- This action was motivated by the relationship $R = -T + B$.
- Setting $F(\phi) = -G(\phi) \rightarrow$ nonminimal coupling to the scalar curvature R.
- Setting $G = 0 \rightarrow$ the same action as before: Teleparallel Dark Energy.
- $F = 1, G = \chi\phi^2$: late-time acceleration without fine tuning.

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Quintom model

- One can generalise this by having two scalar fields which leads to the following quintom models (M. Marciu, Phys. Rev. D **93** (2016) no.12, 123006)

$$S = \int \Big[\frac{R}{2} + \frac{1}{2}\Big(f_1(\phi) + f_2(\sigma)\Big)R + \frac{1}{2}\xi\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\chi\partial_\mu\sigma\partial^\mu\sigma - V(\phi,\sigma) + L_\mathrm{m}\Big] e\, d^4x\,,$$

  where now V depends on two scalar fields σ, ϕ.

- In Marciu 2016: $f_1 = -c_1\phi^2, f_2 = c_2\sigma^2, \chi = -\xi = 1$ (one phantom scalar field+one canonical).
- He studied cosmology using numerical approach and found: late-time DE where Universe evolves towards a Big Rip.

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Generalised Teleparallel Quintom model

- In the same spirit as quintom models, let us propose the following generalised teleparallel action (S. Bahamonde, M. Marciu and P. Rudra, arXiv:1802.09155 [gr-qc]),

Our new theory: Generalised Quintom Teleparallel

$$S = \int \Big[\frac{T}{2} + \frac{1}{2}\Big(f_1(\phi) + f_2(\sigma)\Big)T + \frac{1}{2}\Big(g_1(\phi) + g_2(\sigma)\Big)B + \frac{1}{2}\xi\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\chi\partial_\mu\sigma\partial^\mu\sigma - V(\phi,\sigma) + L_{\rm m}\Big] e\, d^4x\,.$$

- Clearly, this model contains all the other previous theories mentioned (remember again $R = -T + B$).

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Field equations

By varying this action with respect to the tetrads, we get (pure tetrad formalism)

$$\begin{aligned} &2\left(1+f_1(\phi)+f_2(\sigma)\right)\left[e^{-1}\partial_\mu(eS_a{}^{\mu\nu})-E_a^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu}-\frac{1}{4}E_a^\nu T\right] \\ &\quad -E_a^\nu\left[\frac{1}{2}\xi\partial_\mu\phi\partial^\mu\phi+\frac{1}{2}\chi\partial_\mu\sigma\partial^\mu\sigma-V(\phi,\sigma)\right]+\xi E_a^\mu\partial^\nu\phi\partial_\mu\phi \\ &\quad +\chi E_a^\mu\partial^\nu\sigma\partial_\mu\sigma+2\partial_\mu\Big(f_1(\phi)+f_2(\sigma)+g_1(\phi)+g_2(\sigma)\Big)E_a^\rho S_\rho{}^{\mu\nu} \\ &\quad +E_a^\nu\,\Box\left(g_1(\phi)+g_2(\sigma)\right)-E_a^\mu\nabla^\nu\nabla_\mu\left(g_1(\phi)+g_2(\sigma)\right)=\mathcal{T}_a^\nu\,, \end{aligned} \quad (1)$$

where $\Box=\nabla_\alpha\nabla^\alpha;\ \nabla_\alpha$ is the covariant derivative linked with the Levi-Civita connection symbol and $\mathcal{T}_a^\nu$ is the matter energy momentum tensor.

Cosmological equations

- Flat FRW cosmology $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ with tetrads $e^i_\mu = \mathrm{diag}(1, a(t), a(t), a(t))$. Moreover, consider a perfect fluid for matter.
- The modified FRW equations for this theory are:

$$\begin{aligned}
3H^2(1+f_1(\phi)+f_2(\sigma)) &= \rho_{\rm m} + V(\phi,\sigma) + \frac{1}{2}\xi\dot{\phi}^2 \\
&\quad + \frac{1}{2}\chi\dot{\sigma}^2 + 3H(g_1'(\phi)\dot{\phi} + g_2'(\sigma)\dot{\sigma}) , \\
(3H^2+2\dot{H})(1+f_1(\phi)+f_2(\sigma)) &= -p_{\rm m} + V(\phi,\sigma) - \frac{1}{2}\xi\dot{\phi}^2 \\
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- In addition, the scalar field equations:

$$\xi(\ddot{\phi}+3H\dot{\phi})+3H^2 f_1'(\phi)+3g_1'(\phi)\Big(3H^2+\dot{H}\Big)+\frac{V(\phi,\sigma)}{\partial\phi} = 0\,,$$
$$\chi(\ddot{\sigma}+3H\dot{\sigma})+3H^2 f_2'(\sigma)+3g_2'(\sigma)\Big(3H^2+\dot{H}\Big)+\frac{V(\phi,\sigma)}{\partial\sigma} = 0\,.$$

- It is easy to show that the standard conservation equation holds for this theory: $\dot{\rho}_{\rm m}+3H(\rho_{\rm m}+p_{\rm m})=0$.
- Let us assume that the energy potentials are separable:

$$V(\phi,\sigma)=V_1(\phi)+V_2(\sigma)\,.$$

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- In addition, the scalar field equations:

$$\xi(\ddot{\phi} + 3H\dot{\phi}) + 3H^2 f_1'(\phi) + 3g_1'(\phi)\left(3H^2 + \dot{H}\right) + \frac{V(\phi,\sigma)}{\partial\phi} = 0\,,$$
$$\chi(\ddot{\sigma} + 3H\dot{\sigma}) + 3H^2 f_2'(\sigma) + 3g_2'(\sigma)\left(3H^2 + \dot{H}\right) + \frac{V(\phi,\sigma)}{\partial\sigma} = 0\,.$$

- It is easy to show that the standard conservation equation holds for this theory: $\dot{\rho}_{\rm m} + 3H(\rho_{\rm m} + p_{\rm m}) = 0$.
- Let us assume that the energy potentials are separable:

$$V(\phi,\sigma) = V_1(\phi) + V_2(\sigma)\,.$$

The modified FRW equations can be also be rewritten in terms of effective energy and pressure:

$$\begin{aligned} 3H^2 &= \rho_{\rm eff}\,, \\ 3H^2 + 2\dot{H} &= -p_{\rm eff}\,, \end{aligned}$$

where we have defined $\rho_{\rm eff} = \rho_{\rm m} + \rho_\phi + \rho_\sigma, \;\; p_{\rm eff} = p_{\rm m} + p_\phi + p_\sigma$ and

$$\begin{aligned} \rho_\phi &= -3H^2 f_1(\phi) + V_1(\phi) + \frac{1}{2}\xi\dot{\phi}^2 + 3Hg_1'(\phi)\dot{\phi} \\ p_\phi &= (3H^2 + 2\dot{H})f_1(\phi) + \frac{1}{2}\xi\dot{\phi}^2 + 2H\dot{\phi}f_1'(\phi) - \ddot{g}_1 - V_1(\phi)\,, \\ \rho_\sigma &= -3H^2 f_2(\sigma) + V_2(\sigma) + \frac{1}{2}\chi\dot{\sigma}^2 + 3Hg_2'(\sigma)\dot{\sigma}\,, \\ p_\sigma &= (3H^2 + 2\dot{H})f_2(\sigma) + \frac{1}{2}\chi\dot{\sigma}^2 + 2H\dot{\sigma}f_2'(\sigma) - \ddot{g}_2 - V_2(\sigma)\,. \end{aligned}$$

Then we can define the following quantities:

$$w_{\rm de} = \frac{p_\phi + p_\sigma}{\rho_\phi + \rho_\sigma}\,, \tag{2}$$

$$\omega_{\rm eff} = \frac{p_{\rm eff}}{\rho_{\rm eff}} = \frac{p_{\rm m} + p_\phi + p_\sigma}{\rho_{\rm m} + \rho_\phi + \rho_\sigma}\,, \tag{3}$$

$$\Omega_{\rm m} = \frac{\rho_{\rm m}}{3H^2}\,, \tag{4}$$

$$\Omega_\phi = \frac{\rho_\phi}{3H^2}\,, \tag{5}$$

$$\Omega_\sigma = \frac{\rho_\sigma}{3H^2}\,, \tag{6}$$

where $\Omega_{\rm m} + \Omega_\phi + \Omega_\sigma = 1$ holds.

Cosmology: asantz assumed

Let us now assume the following ansatz:

- $f_1(\phi) = c_1\phi^2\,, f_2(\sigma) = c_2\sigma^2\,, g_1(\phi) = c_3\phi^2\,, g_2(\sigma) = c_4\sigma^2$, where c_i ($i = 1, .., 4$) are constants.
- $V(\phi, \sigma) = V_1(\phi) + V_2(\sigma) = V_1 e^{-\lambda_1\phi} + V_2 e^{-\lambda_2\phi}$, where V_1, V_2 and $\lambda_1 > 0, \lambda_2 > 0$ are constants.
- Barotropic fluid: $p_{\rm m} = (\gamma - 1)\rho_{\rm m}$, so we directly find that $\rho_{\rm m} = \rho_0 a(t)^{-3\gamma}$.
- Note that the next results are valid only with those ansantz.

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Dynamical system

- Let us now introduce the dimensionless variables

$$s^2 = \frac{\rho_{\rm m}}{3H^2}, x^2 = \frac{\dot{\phi}^2}{6H^2}, y^2 = \frac{V_1(\phi)}{3H^2}, z = 2\sqrt{6}\xi\phi,$$
$$u^2 = \frac{\dot{\sigma}^2}{6H^2}, v^2 = \frac{V_2(\sigma)}{3H^2}, w = 2\sqrt{6}\chi\sigma\,,$$

  which straightforwardly generalise the normalised variables used to analyse standard quintessence (E. J. Copeland, A. R. Liddle and D. Wands,Phys. Rev. D **57** (1998) 4686)

- Then, the first FRW equation can be written as

$$s^2 = 1 - \xi x^2 - y^2 + \frac{c_1}{24\xi^2}z^2 - \frac{c_3}{\xi}xz - \chi u^2 - v^2 + \frac{c_2}{24\chi^2}w^2 - \frac{c_4}{\chi}uw\,.$$

- We can reduce the dimensionality of the dynamical system from 7 to 6.

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Dynamical system

- By changing $N = \log a$, the 6D dynamical system becomes

$$\frac{dx}{dN} = \frac{1}{\sqrt{6}}\left(q - \sqrt{6}px\right), \tag{7}$$

$$\frac{dy}{dN} = -\frac{y}{2}\left(2p + \sqrt{6}\lambda_1 x\right), \tag{8}$$

$$\frac{dz}{dN} = 12\xi x, \tag{9}$$

$$\frac{du}{dN} = \frac{1}{\sqrt{6}}\left(q - \sqrt{6}pu\right), \tag{10}$$

$$\frac{dv}{dN} = -\frac{v}{2}\left(2p + \sqrt{6}\lambda_2 u\right), \tag{11}$$

$$\frac{dw}{dN} = 12u\chi. \tag{12}$$

Dynamical system

- We have defined the following quantities

$$
\begin{aligned}
p &= -\frac{1}{2\left(\chi^3\left(c_1\xi z^2+6c_3^2z^2+24\xi^3\right)+c_2\xi^3w^2\chi+6c_4^2\xi^3w^2\right)}\Big[3\chi^3\Big(c_1z(4c_3z \\
&\quad +\xi(16\xi x+\gamma z))+4\Big(3c_3^2z^2-c_3\xi\left(6\xi x(4\xi x+(\gamma-2)z)+\sqrt{6}\lambda_1y^2z\right) \\
&\quad -6\xi^3\Big(\gamma\left(u^2\chi+v^2+\xi x^2+y^2-1\right)-2\left(u^2\chi+\xi x^2\right)\Big)\Big)\Big) \\
&\quad +c_2\xi^3w(4c_4w+\chi(16u\chi+\gamma w))+12c_4^2\xi^3w^2-4c_4\xi^3\chi\Big(6u\chi(4u\chi \\
&\quad +(\gamma-2)w)+\sqrt{6}\lambda_2v^2w\Big)\Big]\,, \\
q &= -\frac{\sqrt{6}c_1z+\sqrt{6}c_3(p+3)z+6\xi\left(\sqrt{6}\xi x-\lambda_1y^2\right)}{2\xi^2}\,, \\
\lambda_1 &= -\frac{V_1'(\phi)}{V_1(\phi)}\,, \\
\lambda_2 &= -\frac{V_2'(\sigma)}{V_2(\sigma)}\,.
\end{aligned}
$$

Dynamical system: analysis

- There are 21 critical points for the dynamical system, but only 13 satisfy $y \geq 0$ and $v \geq 0$ which ensures that the potentials are positive:

Point	x	y	z	u	v	w
O	0	0	0	0	0	0
$A_\pm$	0	0	0	0	0	$\pm 2\sqrt{-\frac{6}{c_2}\kappa\chi}$
$B_\pm$	0	0	$\pm 2\sqrt{-\frac{6}{c_1}\kappa\xi}$	0	0	0
$C_\pm$	0	0	0	0	$\frac{\sqrt{2(c_2+3c_4)}\sqrt{\chi(c_2+3c_4)\pm\Delta_1}}{\sqrt{c_2\chi}\lambda_2}$	$\frac{2\sqrt{6}(\chi(c_2+3c_4)\pm\Delta_1)}{c_2\lambda_2}$
$D_\pm$	0	$\frac{\sqrt{2(c_1+3c_3)}\sqrt{c_1\xi+3c_3\xi\pm\Delta_2}}{\sqrt{c_1\xi}\lambda_1}$	$\frac{2\sqrt{6}(c_1\xi+3c_3\xi\pm\Delta_2)}{c_1\lambda_1}$	0	0	0
$E_\pm$	0	$\sqrt{\frac{c_1+3c_3}{\sqrt{6}\lambda_1\xi}z}$	z	0	$\frac{\sqrt{c_2+3c_4}\sqrt{12c_2\lambda_1\xi^2\chi+36c_4\lambda_1\xi^2\chi\pm\Delta_3\sqrt{\lambda_1}\xi\chi}}{\sqrt{6\chi c_2\lambda_1}\lambda_2\xi}$	$\frac{\chi\left(12\sqrt{\lambda_1}\xi(c_2+3c_4)\pm\Delta_3\right)}{\sqrt{6\lambda_1}c_2\lambda_2\xi}$
$F_\pm$	0	$\sqrt{\frac{c_1+3c_3}{\sqrt{6}\lambda_1\xi}z}$	z	0	$-\frac{\sqrt{c_2+3c_4}\sqrt{12c_2\lambda_1\xi^2\chi+36c_4\lambda_1\xi^2\chi\pm\Delta_3\sqrt{\lambda_1}\xi\chi}}{\sqrt{6\chi c_2\lambda_1}\lambda_2\xi}$	$\frac{\chi\left(12\sqrt{\lambda_1}\xi(c_2+3c_4)\pm\Delta_3\right)}{\sqrt{6\lambda_1}c_2\lambda_2\xi}$

Dynamical system: analysis

Point	Existence	Acceleration	Stability
O	Always	$\gamma < -1/3$	Saddle point
$A_\pm$	$c_2 < 0$	$c_4 > 0 \wedge c_2 < -2c_4$	Stable if $c_4 > 0 \wedge c_2 < -3c_4 \wedge c_1 > \frac{c_2 c_3}{c_4} \wedge \xi \geq \frac{24c_4(c_1 c_4 - c_2 c_3)}{c_2^2}$ Unstable if $c_4 < 0 \wedge c_2 < 0 \wedge c_1 > \frac{c_2 c_3}{c_4} \wedge \xi \geq \frac{24c_4(c_1 c_4 - c_2 c_3)}{c_2^2}$
$B_\pm$	$c_1 < 0$	$c_3 > 0 \wedge c_1 < -2c_3$	Stable if $c_3 > 0 \wedge c_1 < -3c_3 \wedge c_2 > \frac{c_1 c_4}{c_3} \wedge \chi \geq \frac{24c_3(c_2 c_3 - c_1 c_4)}{c_1^2}$ Unstable if $c_3 < 0 \wedge c_1 < 0 \wedge c_2 > \frac{c_1 c_4}{c_3} \wedge \chi \geq \frac{24c_3(c_2 c_3 - c_1 c_4)}{c_1^2}$

Table: Existence, acceleration condition and stability for the critical points O, $A_\pm$ and $B_\pm$

Dynamical system: analysis

- The point O is always a saddle point and it represents a matter dominated era.
- Critical points A_+ and A_- correspond to a dynamical scenario where the first quintom field ϕ is absent, whereas the second quintom field σ is frozen, without any kinetic or potential energy. DE dominated points.
- Critical points $B_\pm$ is similar as $A_\pm$ but the other scalar field is frozen.
- All the remaining points are non-hyperbolic, hence, linear stability fails! All those points represent dark energy dominated universe (see paper for more details).

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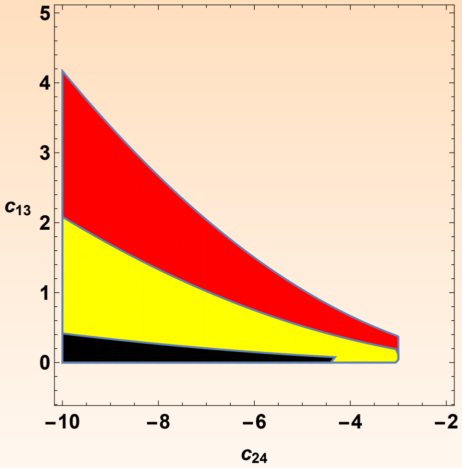

Figure: Regions where the points $A_{\pm}$ are stable. The black, yellow and red regions represent the cases where $\xi = 0.1, 0.5$ and 1 respectively. Here: $c_{24} = c_2/c_4$ and $c_{13} = c_1 - c_3 c_{24}$.

Numerical analysis: quintom coupled only with T

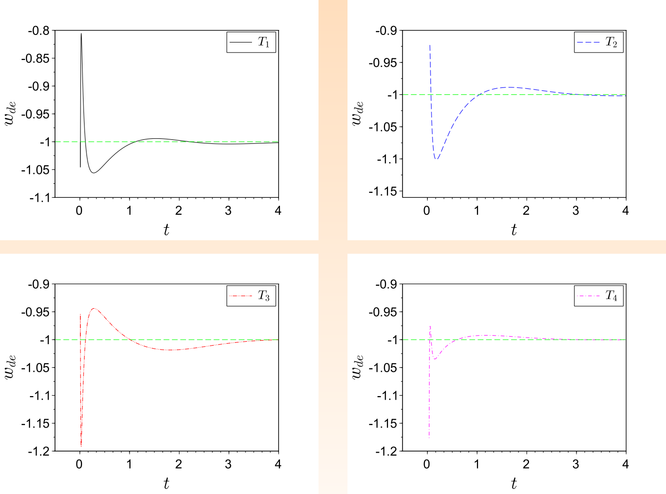

Figure: The evolution of the dark energy equation of state for scalar torsion coupling models T_1, T_2, T_3, T_4

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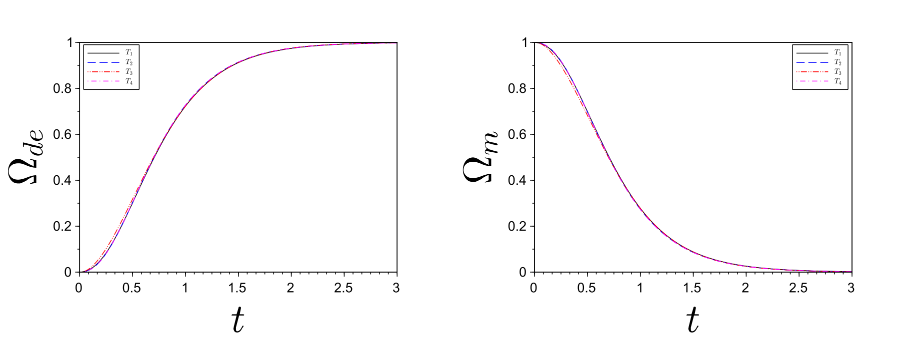

Figure: The evolution of the quintom energy density and matter energy density for scalar torsion coupling models

Outline

Conclusions

- We introduced a new quintom (two scalar field) nonminimally coupled model with both a torsion scalar T and a boundary term B (which is connected with scalar curvature via $R = -T + B$)
- We analysed the dynamical system which is a 6D one with 13 physical critical points. One critical point is a matter saddle point, the others energy dominated eras. We classified them and analysed (see paper for more details).
- See Big new review in DS in cosmology: S. Bahamonde, C. G. Boehmer, S. Carloni, E. J. Copeland, W. Fang and N. Tamanini, "Dynamical systems applied to cosmology: dark energy and modified gravity," arXiv:1712.03107 [gr-qc] .

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Conclusions

- Numerical study: early time matter dominated (scalar field almost frozen) which evolves towards a De Sitter in the distant future.
- Although the late time dynamics of our model corresponds to a ΛCDM evolution, in the early times, the dark energy equation of state presents a quintom behaviour which might be in favor with the recent observations which suggest that the phantom divide line was crossed in the near past (crossing phantom divide line).

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