

Teleparallel gravity and its modifications

Sebastián Bahamonde

sebastian.beltran.14@ucl.ac.uk

Department of Mathematics, University College London.

Revising Foundations of Physics, UCL

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Outline

- 1 Introduction to Teleparallel equivalent of general relativity
 - Basic concepts in teleparallel gravity
 - Gauge structure: translation group
 - Torsion, connection and action
 - Teleparallel gravity vs General Relativity
- 2 Modified teleparallel gravity theories
 - General properties
 - Modified teleparallel theories of gravity: $f(T, B)$ gravity
 - Modified teleparallel theories of gravity: $f(T, B, T_G, B_G)$
- 3 Conclusions

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Tetrad fields

- Assuming that the manifold is differentiable: Define tetrads (or vierbein) $\{e_a\}$ (or $\{e^a\}$) which are the linear basis on the spacetime manifold.
- At each point of the spacetime, tetrads gives us basis for vectors on the tangent space.
- Notation: Greek letters \rightarrow space-time indices; Latin letters \rightarrow tangent space indices; $E_a{}^\mu$ is the inverse of the tetrad.
- Then for example $e_a = E_a{}^\mu dx_\mu$ and $e^a = e^a{}_\mu dx^\mu$.
- Tetrads satisfy the orthogonality condition: $E_m{}^\mu e^n{}_\mu = \delta_m^n$ and $E_m{}^\nu e^m{}_\mu = \delta_\mu^\nu$

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Tetrads and Special Relativity

- **General basis $\{e_a\}$ satisfies $[e_a, e_b] = f^c{}_{ab}e_c$, where $f^c{}_{ab} = E_a^\mu E_b^\nu (\partial_\nu e^c{}_\mu - \partial_\nu e^c{}_\nu)$ is the coefficient of anholonomy (curl of the basis).**
- Inertial frame (or holonomic frame) e'_a : $f'^a{}_{cd} = 0$ (global property)
- Special Relativity in Minkowski: Linear frames (trivial frames) gives us a relation between η_{ab} (tangent) and $\eta_{\mu\nu}$ (spacetime): $\eta_{ab} = \eta_{\mu\nu} E_a^\mu E_b^\nu$ and $\eta_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$.
- In trivial frames: Curvature and torsion are zero.

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- SR: $e'^a{}_{\mu} = \partial_{\mu}x'^a$ and then $x^a \rightarrow \Lambda^a{}_b x^b$. The new frame will be

Lorentz-rotated frame in SR

$$e^a{}_{\mu} = \Lambda^a{}_b e'^b{}_{\mu} = \partial_{\mu}x^a + w^a{}_{b\mu}x^b \equiv D_{\mu}x^a, \quad (1)$$

where $w^a{}_{b\mu} = \Lambda^a{}_e \partial_{\mu} \Lambda_b{}^e$ is known as the spin connection which represents inertial effects.

- Example free particles: inertial frame: $\frac{du'^a}{d\lambda} = 0$. This Eq. is not local Lorentz invariant
- However, in the anholonomic frame, changing $e^a{}_{\mu} = \Lambda^a{}_b e'^b{}_{\mu}$, one gets $\frac{du'^a}{d\lambda} + w^a{}_{b\mu}u^b w^{\mu} = 0$ which is Lorentz covariant!
- Spin-connection in SR represent inertial effects and they are very important to the local Lorentz invariance.

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Gauge theory of translations - Teleparallel gravity

- TG is a gauge theory of translations: a field ψ transforms covariantly under a translation gauge $x^a \rightarrow x'^a + \xi^a(x^\mu)$ if a gauge potential $B^a{}_\mu = -\partial_\mu \xi^a$ is introduced.
- The translational gauge covariant derivative can be written as $e_\mu \psi = e^a{}_\mu \partial_a \psi$, and then

$$e^a{}_\mu = \partial_\mu x^a + B^a{}_\mu, \quad (2)$$

where $e^a{}_\mu$ cannot be a trivial frame.

- This means that one needs to replace a Minkowski metric to a Riemannian metric giving us

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu, \quad g^{\mu\nu} = \eta^{ab} E^\mu{}_a E^\nu{}_b. \quad (3)$$

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Gauge theory of translations - Inertial effects

- Let us now consider inertial effects in TG.
- By performing a Lorentz transformation $x^a \rightarrow \Lambda^a_b x^b$ one finds that the tetrad becomes

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Translation Field Strength

- Like any gauge theory, the field strength is the covariant rotational of the gauge potential

$$T^a{}_{\mu\nu} = D_\mu B^a{}_\nu - D_\nu B^a{}_\mu. \quad (5)$$

- Since $e^a{}_\mu = D_\mu x^a + B^a{}_\mu$, one can rewrite the field strength as

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- This expression coincides with the Torsion tensor!
- It can be proved that the torsion tensor transforms covariantly under both diffeomorphisms and local Lorentz transformations and only represent gravitational effects.

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Connection in Teleparallel gravity

- Let us now introduce the so-called “Weitzenböck connection”:

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$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_{\mu} e^a{}_{\nu} = E_a{}^{\rho} (\partial_{\mu} e^a{}_{\nu} + w^a{}_{b\mu} e^b{}_{\nu}). \quad (7)$$

- By using this connection, one can express the torsion tensor as follows

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$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}. \quad (8)$$

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Connection in TG

- The Weitzenböck connection $\tilde{\Gamma}^\rho_{\nu\mu}$ is related to the Levi-Civita connection $\Gamma^\rho_{\nu\mu}$ via

Relationship between connections

$$\tilde{\Gamma}^\rho_{\nu\mu} = \Gamma^\rho_{\nu\mu} + K^\rho_{\mu\nu}, \quad (9)$$

where $K^\rho_{\mu\nu} = \frac{1}{2}(T_\mu^\rho{}_\nu + T_\nu^\rho{}_\mu - T^\rho{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

Curvature in Teleparallel gravity

$$R^a{}_{b\mu\nu}(\omega^a{}_{b\mu}) = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu} \equiv 0. \quad (10)$$

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$$R^a{}_{b\mu\nu}(\omega^a{}_{b\mu}) = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\omega^c{}_{b\nu} - \omega^a{}_{c\nu}\omega^c{}_{b\mu} \equiv 0. \quad (10)$$

Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar T

$$S_{\text{TG}} = \int [-T + 2\kappa^2 \mathcal{L}_m] e d^4x. \quad (11)$$

where $\kappa^2 = 8\pi G$, $e = \det(e_a^\mu) = \sqrt{-g}$, \mathcal{L}_m matter Lagrangian and $T = \frac{1}{4}T^\rho{}_{\mu\nu}T^{\mu\nu}{}_\rho + \frac{1}{2}T^\rho{}_{\mu\nu}T^{\nu\mu}{}_\rho - T^\lambda{}_{\lambda\mu}T_\nu{}^{\nu\mu}$.

- T and the scalar curvature R differs by a boundary term B as $R = -T + B$ so:

Equivalence between field equations

The teleparallel field equations are equivalent to the Einstein field equations.

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Teleparallel gravity vs General Relativity

Two completely equivalent ways of understanding gravity:

Connections and strength fields

G.R. \implies Levi-Civita connection \implies Curvature with vanishing torsion

TG \implies Weitzenböck connection \implies Torsion with vanishing curvature (flat).

How gravity is explained in both theories?

GR \implies Geometry (curvature of space-time) \implies geodesic equations

TG \implies Forces \implies Force equations as Maxwell eqs (no geodesic eq.).

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GR \implies NO (only diffeomorphism)

TG \implies Gauge theory of the translations

Must have the equivalence principle?

GR \implies YES

TG \implies Can survive with or without

Can we separate inertia with gravity?

GR \implies NO (mixed) \implies No tensorial expression for the gravitational energy-momentum density

TG \implies YES \implies gravitational energy-momentum density is a tensor.

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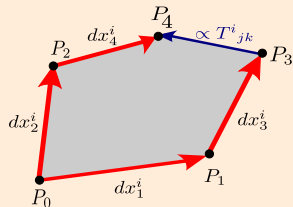
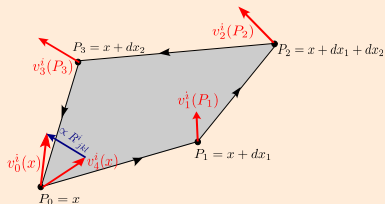
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Geometrical differences

Curvature \implies how the tangent spaces roll along the curve.

Torsion \implies how tangent spaces twist about a curve when they are parallel transported



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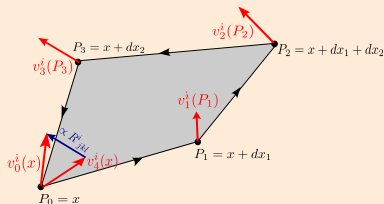


Figure: Transporting a vector in a closed trajectory creates a different vector

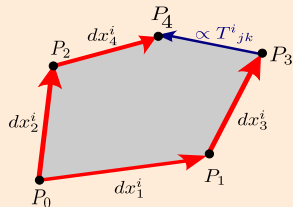


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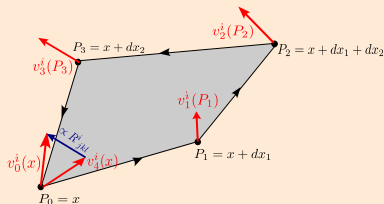


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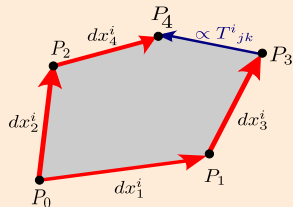


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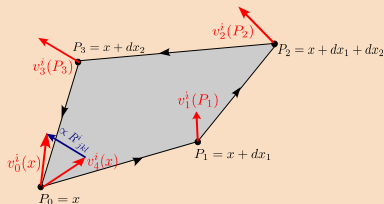


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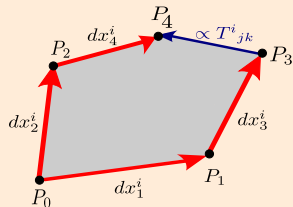


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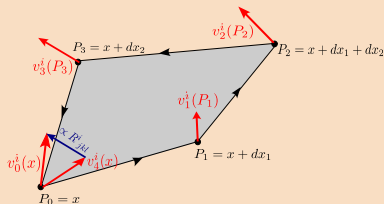


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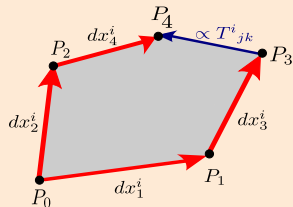


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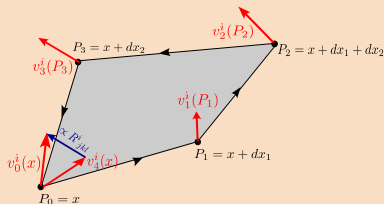


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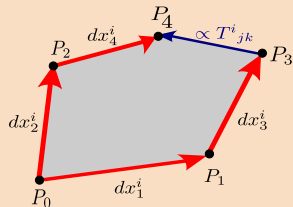


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Pure tetrad teleparallel

- When it is assumed that for all frames the spin connection is zero, $w^a{}_{bc} = 0$, then one refers to pure tetrad teleparallel theories. In this case, the torsion tensor becomes

$$T^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu = f^a{}_{\mu\nu} \quad (12)$$

- Therefore, the torsion tensor and the scalar torsion T are not invariant under Local Lorentz transformations.
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Modified teleparallel theories of gravity

- As there are many modifications in GR, it is also possible to construct modifications to teleparallel gravity.
- If one modifies pure tetrad teleparallel gravity, one finds theories which are not invariant under local LT.
- Two approaches for constructing modified teleparallel theories:

How to construct modified teleparallel theories

- Both approaches give the same field equations but (1) is more theoretically correct. However, there is still a debate about how to find $w^a{}_{bc}$.
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Motivating from GR: $f(R)$ gravity

- A well studied modification of GR is $f(R)$ gravity, which has the following action (change $R \rightarrow f(R)$)

$f(R)$ gravity action

$$S_{f(R)} = \int (f(R) + 2\kappa^2 \mathcal{L}_m) \sqrt{-g} d^4x .$$

- Here, f is an arbitrary (sufficiently smooth) function of the Ricci scalar.
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- Very famous and explore theory which mimics cosmological observations.

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Relationship between R and T

$$R = -T + \frac{2}{e} \partial_\mu (e T^\mu) = -T + B.$$

- Inspired by the later discussion, we define the action
(Bahamonde et. al Phys.Rev. D92,10, 104042 (2015))

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- It is possible to show that this theory has the correct limits:

Limiting cases

- ① When $f(T, B) = f(T)$, one obtains $f(T)$ gravity
- ② When $f(T, B) = f(-T + B) = f(R)$, one obtains the teleparallel equivalent of $f(R)$ gravity
- ③ Other new theories such as $f(T, B) = T + F(B)$.

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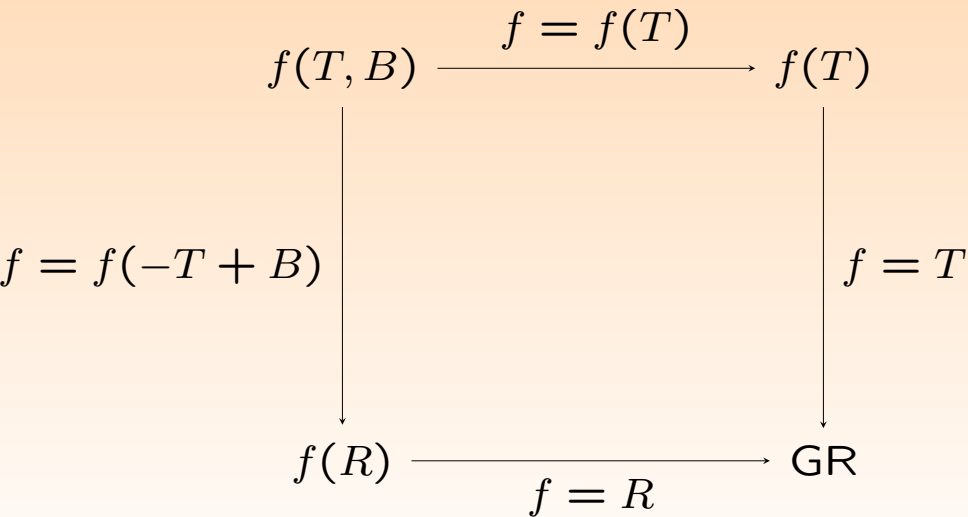
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Outline

- 1 Introduction to Teleparallel equivalent of general relativity
 - Basic concepts in teleparallel gravity
 - Gauge structure: translation group
 - Torsion, connection and action
 - Teleparallel gravity vs General Relativity
- 2 Modified teleparallel gravity theories
 - General properties
 - Modified teleparallel theories of gravity: $f(T, B)$ gravity
 - Modified teleparallel theories of gravity: $f(T, B, T_G, B_G)$
- 3 Conclusions

Gauss-Bonnet extension

- The Gauss-Bonnet term is a quadratic combination of the Riemann tensor and its contractions given by

Gauss-Bonnet term

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}.$$

- This term can be expressed in a similar way as before which simply reads as

Relationship between Gauss-Bonnet G and T_G and B_G

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- Inspired by the later discussion, we define the action
(Bahamonde et. al, Eur.Phys.J. C76 (2016) no.10, 578)

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$$S_{f(T,B,T_G,B_G)} = \int [f(T, B, T_G, B_G) + 2\kappa^2 \mathcal{L}_m] e d^4x .$$

- This action is very general. For instance, we can recover $f = f(T, T_G)$ gravity or $f(-T + B, -T_G + B_G) = f(R, G)$ gravity.
- In Bahamonde et. al, Eur.Phys.J. C76 (2016) no.10, 578 , an additional extension where the trace of the energy-momentum tensor $\mathcal{T} = E_a^\beta \mathcal{T}_\beta^a$ was introduced too; $f(T, B, T_G, B_G, \mathcal{T})$.

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Good tetrads: Example in FRWL Cosmology with $k = +1$

- Since T, B, T_G and B_G are not invariant under local Lorentz transformations \implies One needs to be careful with the choice of the tetrad.
- The simplest tetrad field which yields the FRWL metric $ds^2 = -dt^2 + a(t)^2 \left[\frac{1}{1-kr^2} dr^2 + d\Omega^2 \right]$ is a diagonal one given by

$$e_{\mu}^a = \text{diag} \left(1, a(t)/\sqrt{1-kr^2}, a(t)r, a(t)r \sin \theta \right).$$

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$$\Lambda^a{}_b = \begin{pmatrix} 1 & 0 \\ 0 & \mathcal{R}(\alpha, \beta, \gamma) \end{pmatrix},$$

is the local Lorentz transformation matrix. Here, R is the 3-dimensional rotation in the tangent space parametrised by three Euler angles $\alpha = \theta - \frac{\pi}{2}$, $\beta = \phi$ and $\gamma = \gamma(r)$.

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- Using the rotated tetrad, the torsion scalar and the boundary term are

$$T = -\frac{4}{a^2} \left(\frac{\sqrt{1 - kr^2}}{r^2} \left[r\gamma' \cos \gamma + \sin \gamma \right] + \frac{1}{r^2} \right) + 6 \frac{\dot{a}^2}{a^2} + 2 \frac{k}{a^2},$$

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- For the closed case $k = +1$, we need to set $\gamma(r) = -\operatorname{arcsinh}(\sqrt{1 + r^2})$ to have T and B position independent (also T_G and B_G) giving us in this frame:

$$T = 6H^2 + \frac{2}{a^2}, B = 6(H^2 + 2\dot{H}) + \frac{8}{a^2}, T_G = -\frac{24\ddot{a}}{a}(H^2 - \frac{1}{a^2}), B_G = \frac{48\ddot{a}}{a^3}, \quad (13)$$

where $H = \dot{a}/a$ is the Hubble parameter. Clearly,
 $R = -T + B = 6(H^2 + 2\dot{H} + 1/a^2)$ and
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$f(T, B, T_G, B_G)$ diagram

$$\begin{array}{ccccc} f(T, B, T_G, B_G) & \xrightarrow{f = f(T, T_G)} & f(T, T_G) & \xrightarrow{f = f(T)} & f(T) \\ \downarrow f = f(-T + B, -T_G + B_G) & & & & \downarrow f = T \\ f(R, G) & \xrightarrow{f = f(-T + B)} & f(R) & \xrightarrow{f = R} & GR \end{array}$$

Conclusions

- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero curvature and non-zero torsion (Weitzbröck connection).
- TG is equivalent on its field equations to GR, but their physical interpretation are different.
- Modified teleparallel theories (in the pure tetrad formalism) are not invariant under Lorentz transformations but one can solve this problem:
 - Starting with a non-pure tetrad formalism with spin connection being different than zero.
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