# Nonlocal Teleparallel Gravity

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# Short Term Scientific Missions

- The work that will be presented here was developed in one of my STMS held in Universitá degli Studi di Napoli Federico II", Naples, Italy (March 2017).
- Two papers were published from this visit
  - S. Bahamonde, S. Capozziello, M. Faizal and R. C. Nunes, 'Nonlocal Teleparallel Cosmology," Eur. Phys. J. C **77** (2017) no.9, 628
  - S. Bahamonde, S. Capozziello and K. F. Dialektopoulos, "Constraining Generalized Non-local Cosmology from Noether Symmetries," Eur. Phys. J. C **77** (2017) no.11, 722.

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# Outline

# Connection in Teleparallel gravity

- Teleparallel gravity (TEGR) is an alternative formulation of gravity which uses tetrads as the dynamical variables.
- Let us introduce the so-called "Weitzenböck connection":

**Weitzenböck connection**

$$\tilde{\Gamma}^{\rho}{}_{\mu\nu} = E_a{}^{\rho} D_\mu e^a{}_\nu = E_a{}^{\rho}(\partial_\mu e^a{}_\nu + w^a{}_{b\mu} e^b{}_\nu)\,.$$

- By using this connection, one can express the torsion tensor as follows

**Torsion tensor**

$$T^{\rho}{}_{\mu\nu} = \tilde{\Gamma}^{\rho}{}_{\nu\mu} - \tilde{\Gamma}^{\rho}{}_{\mu\nu}\,.$$

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**Relationship between connections**

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where $K^{\rho}{}_{\mu\nu} = \frac{1}{2}(T_{\mu}{}^{\rho}{}_{\nu} + T_{\nu}{}^{\rho}{}_{\mu} - T^{\rho}{}_{\mu\nu})$ is the contorsion tensor.

- In this connection, it is easy to verify that the spacetime is globally flat:

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## Curvature in Teleparallel gravity

$$R^{a}{}_{b\mu\nu}(\omega^{a}{}_{b\mu}) = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\omega^{c}{}_{b\nu} - \omega^{a}{}_{c\nu}\omega^{c}{}_{b\mu} \equiv 0\,.$$

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# Teleparallel action

- The teleparallel action is formulated based on a gravitational scalar called the torsion scalar $T$

$$S_{\mathrm{TEGR}} = \int \left[T + 2\kappa^2 \mathcal{L}_{\mathrm{m}}\right] e\, d^4x\,.$$

where $\kappa^2 = 8\pi G$, $e = \det(e_a^{\mu}) = \sqrt{-g}$, $\mathcal{L}_{\mathrm{m}}$ matter Lagrangian and $T = \frac{1}{4}T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + \frac{1}{2}T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} - T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}$.

- $T$ and the scalar curvature $R$ differs by a boundary term $B$ as $R = -T + B$ so:

**Equivalence between field equations**

The teleparallel field equations are equivalent to the Einstein field equations.

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# Two different ways of understanding gravity

## Equivalence on their field equations

VERY IMPORTANT POINT: TEGR has the same equations as GR, so **CLASSICALLY** it is impossible to make any observation to distinguish between them.

## Validity of TEGR

VERY IMPORTANT POINT: All classical experiments already done, that have confirmed GR, also can be understood as a confirmation of TEGR.

## What to do?

Since both theories predict the same classical experiments, but they are different conceptually and physically, how can we know which theory is the correct one?

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# Nonlocality

- Since TEGR and GR are conceptually different, they are expected to produce different quantum effects.
- Many quantum gravity proposals (string theory, loop quantum gravity) $\Rightarrow$ Intrinsic extended structure in the geometry of spacetime $\Rightarrow$ Effective nonlocal behavior for spacetime.
- Then, first order corrections from quantum gravity might produce nonlocal deformations of GR.
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## Non-locality - A way to contrast GR with TEGR

- Quantum corrected nonlocal GR can be written as

**Nonlocal Quantum correction action**

$$\mathcal{S}_1 = \mathcal{S}_{\rm GR} + \mathcal{S}_{\rm GRNL}.$$

- Similarly, in TEGR, one can consider

**Teleparallel Nonlocal Quantum correction action**

$$\mathcal{S}_2 = \mathcal{S}_{\rm TEGR} + \mathcal{S}_{\rm TEGRNL}.$$

- It is not possible to differentiate classically between $\mathcal{S}_{\rm GR}$ and $\mathcal{S}_{\rm TEGR}$, but the quantum corrections to these theories $\mathcal{S}_{\rm GRNL}$ and $\mathcal{S}_{\rm TEGRNL}$ are different.
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# Deser-Woodard Nonlocal Gravity

- There exists different nonlocal models based on standard GR. One very interesting one has the following action (S. Deser and R. P. Woodard, Phys. Rev. Lett. **99** (2007) 111301)

**DW Nonlocal action**

$$\mathcal{S} = \mathcal{S}_{\rm GR} + \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\,Rf\Big((\Box^{-1}R)\Big) + \mathcal{S}_{\rm m}\,.$$

- Here $f$ is an arbitrary function which depends on the retarded Green function evaluated at the curvature scalar $R$ and $\mathcal{G}[f](x)$ is a nonlocal operator which can be written in terms of the Green function $G(x,x')$ as

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# Deser-Woodard Nonlocal Gravity - Properties

- It has been shown that DW NG in its **standard form**: (R. P. Woodard, Found. Phys. **44** (2014) 213):
  - Ghost-free and stable ($f$ must satisfy a condition).
  - Does not propagate extra degrees of freedom.
  - Can mimic dark energy without a $\Lambda$ (specific distortion function).
  - It is consistent with Solar system constraints.
  - Acasual (due to the advanced Green function).
- It is possible to **localised the action** by introducing two auxiliary field $\phi = \Box^{-1}R$ and $\xi = -f'(\phi)R$, which gives a causal theory but contains ghost (can be avoided for some $f$ at some very special times). (S. Nojiri and S. D. Odintsov (2008), S. Nojiri, S. D. Odintsov, M. Sasaki and Y. I. Zhang (2011))

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# Nonlocal Teleparallel gravity - Why study this in TEGR?

- TEGR **DOES NOT REQUIRE** the equivalence principle, and it has been argued that quantum effects can cause the violation of the Equivalence Principle.
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# Nonlocal Teleparallel Gravity

- Inspired by DS nonlocal model, let us propose the following action (SB, M Faizal,S. Capozziello, R.N.Nunes Eur.Phys.J (2017))

**Teleparallel nonlocal action**

$$\mathcal{S} = \mathcal{S}_{\text{TEGR}} + \frac{1}{2\kappa}\int d^4x\, e\, T f\Big((\Box^{-1}T)\Big) + S_{\text{m}}\,,$$

where now $f$ is a function of the Green function evaluated at the torsion scalar $T$.

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# Nonlocal Teleparallel Gravity

- The field equations for the latter action are difficult to handle, but one can use a trick by introducing two auxiliary fields $\phi$ and $\theta$ to rewrite the nonlocal action as

$$\mathcal{S} = \frac{1}{2\kappa}\int d^4x\, e\Big[T(f(\phi)-1) - \partial_\mu\theta\partial^\mu\phi - \theta T\Big] + S_{\rm m}\,.$$

  where now $\phi = \Box^{-1}T$ and $\Box\theta = -f'(\phi)T$.
- To avoid ghosts, those functions must satisfy

**Ghost-free conditions**

$$f'(\phi) > \frac{1-f-\theta}{6} > 0$$

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# Nonlocal Teleparallel Cosmology

- Future experiments probing nonlocal effects could be used to test whether General Relativity or Teleparallel Gravity give the most consistent picture of gravitational interaction (Weak Equivalence Principle or photon time delay and gravitational red shifts measured by high energy gamma ray).
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# Generalised nonlocal gravity

- In the next study, we generalise the action to (S. Bahamonde, S. Capozziello and K. F. Dialektopoulos, Eur. Phys. J. C **77** (2017) no.11, 722)

$$\mathcal{S} = \mathcal{S}_{\rm TEGR} + \frac{1}{2\kappa}\int d^4x\, e\,(\xi T + \chi B) f\Big((\Box^{-1}T), (\Box^{-1}B)\Big) + \mathcal{S}_{\rm m}\,.$$

  where $B$ is the boundary term which relates the Ricci scalar $R$ with $T$ via $R = -T + B$. Here $\xi$ and $\chi$ are constants.
- One can recover Deser-Woodard theory (couplings like $Rf(\Box^{-1}R)$) or other couplings such as $Bf(\Box^{-1}B)$.
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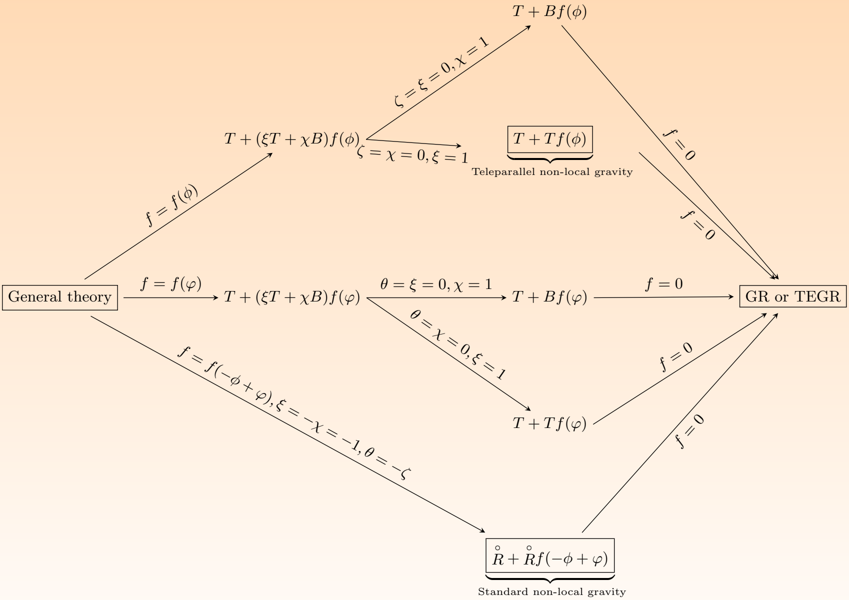


**Figure:** Different theories with four auxiliary fields: $\varphi = \Box^{-1} B$ and $\phi = \Box^{-1} T$, $\Box\theta = (\xi T + \chi B)\frac{\partial f(\phi,\varphi)}{\partial \phi}$, $\Box\zeta = (\xi T + \chi B)\frac{\partial f(\phi,\varphi)}{\partial \varphi}$.

# Generalised nonlocal Cosmology

- We used the Noether symmetry approach for flat FLRW cosmology in order to get analytical cosmological solutions for some nonlocal theories.
- We found an interesting result: from the symmetries of the Lagrangian, we found that the functions $f(\phi,\varphi)$ and $g(\phi,\varphi)$ are either exponential type of couplings (such as $Te^{\Box^{-1}T}$) or linear couplings (such as $T(c_1+c_2\Box^{-1}T)$). This was found without imposing anything by hand.
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# Conclusions

- Teleparallel gravity is a gauge theory of the translation group which leads a special connection with zero cuvature and non-zero torsion (Weitzbröck connection). Its field equations are the same as GR.
- We formulated the first quantum approach to TEGR, considering non-locality which is consistent with cosmological observations.
- Classically, **TEGR and GR are equivalent on their field equations**, but their **nonlocal corrections are different**.
- We then generalised our model for more non-local theories. We found cosmological solutions and also we got that exponentials (or lineal) couplings appeared naturally by Noether symmetries.

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