New black hole solutions with a dynamical traceless nonmetricity tensor in Metric-Affine Gravity

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Fundamental variables and characteristic tensors

- In the most general metric-affine setting, the fundamental variables are a **metric** $g_{\mu\nu}$ (10 comp.) as well as the coefficients $\tilde{\Gamma}^{\rho}{}_{\mu\nu}$ (64 comp.) of an **affine connection**.

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Curvature decomposition, torsion and nonmetricity

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|}N^{\lambda}{}_{\rho|\nu]} + 2N^{\lambda}{}_{\sigma[\mu|}N^{\sigma}{}_{\rho|\nu]} ,$$

$$\tilde{T}^{\mu}{}_{\nu\rho} = \tilde{\Gamma}^{\mu}{}_{\rho\nu} - \tilde{\Gamma}^{\mu}{}_{\nu\rho} ,$$

$$\tilde{Q}_{\mu\nu\rho} = \tilde{\nabla}_{\mu} g_{\nu\rho} = \partial_{\mu} g_{\nu\rho} - \tilde{\Gamma}^{\sigma}{}_{\nu\mu} g_{\sigma\rho} - \tilde{\Gamma}^{\sigma}{}_{\rho\mu} g_{\nu\sigma} .$$

Tildes=General, nothing=Riemannian $\rightarrow$ ∇_{μ}(Levi-Civita), $\tilde{\nabla}_{\mu}$(General)

Decomposition into irreducible parts

- Irreducible decomposition of the torsion tensor:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,,$$

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Dynamics of metric-affine geometry

- Gravitational action with dynamical torsion and nonmetricity:

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_m - \frac{1}{16\pi} \mathcal{L}_g(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q}) \right] .$$

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- Correspondence between geometry and matter:

$$\begin{aligned} \frac{\delta S_g}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu , \\ \frac{\delta S_g}{\delta \omega^a{}_{b\nu}} &= 16\pi\Delta_a{}^{b\nu} . \end{aligned}$$

Here $\theta_a{}^\nu$ is the energy-momentum tensor (canonical) and $\Delta_a{}^{b\nu}$ is the hypermomentum density tensor.

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- Three independent contractions of the curvature tensor and only one independent scalar curvature:

$$\tilde{R}_{\mu\nu} = \tilde{R}^\lambda{}_{\mu\lambda\nu} , \quad \hat{R}_{\mu\nu} = \tilde{R}_\mu{}^\lambda{}_{\nu\lambda} , \quad \tilde{R}^\lambda{}_{\lambda\mu\nu} = 4\nabla_{[\nu}W_{\mu]} ,$$
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- $GL(4,R)$ group allows the definition of a large number of scalars.

MAG theory with shears

- Let us first consider a simple model where torsion is not propagating and the traceless part of nonmetricity is dynamical:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big[-R + 2f_1\tilde{R}_{(\lambda\rho)\mu\nu}\tilde{R}^{(\lambda\rho)\mu\nu} + 2f_2\left(\tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)}\right)\left(\tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)}\right)\Big]\,,$$

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$$+\,2f_2\left(\tilde{R}_{(\mu\nu)}-\hat{R}_{(\mu\nu)}\right)\left(\tilde{R}^{(\mu\nu)}-\hat{R}^{(\mu\nu)}\right)\Big]\,,$$

- As can be seen, the propagation of the nonmetricity field described in the action is carried out by the symmetric part of the curvature tensor and its symmetric contraction:

$$\tilde{R}^{(\lambda\rho)}{}_{\mu\nu}=\tilde{\nabla}_{[\nu}Q_{\mu]}{}^{\lambda\rho}+\frac{1}{2}T^{\sigma}{}_{\mu\nu}Q_{\sigma}{}^{\lambda\rho}\,,$$
$$\tilde{R}_{(\mu\nu)}-\hat{R}_{(\mu\nu)}=\tilde{\nabla}_{(\mu}Q^{\lambda}{}_{\nu)\lambda}-\tilde{\nabla}_{\lambda}Q_{(\mu\nu)}{}^{\lambda}-Q^{\lambda\rho}{}_{\lambda}Q_{(\mu\nu)\rho}+Q_{\lambda\rho(\mu}Q_{\nu)}{}^{\lambda\rho}$$
$$+\,T_{\lambda\rho(\mu}Q^{\lambda\rho}{}_{\nu)}\,,$$

which in turn constitute deviations from the third Bianchi of GR.

Spherical symmetry in metric-affine geometry

- Metric, torsion and nonmetricity tensors in symmetric space-times:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q^\lambda{}_{\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Killing vectors in static and spherically symmetric space-times:

$$\begin{aligned}
\eta_0 &= \partial_t\,,\\
\xi_1 &= \sin\varphi\,\partial_\vartheta + \cot\vartheta\cos\varphi\,\partial_\varphi\,,\\
\xi_2 &= -\cos\varphi\,\partial_\vartheta + \cot\vartheta\sin\varphi\,\partial_\varphi\,,\\
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- Metric:

$$\#10\,\mathrm{dof} \to \#2\,\mathrm{dof}\left\{ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin\vartheta^2 d\varphi^2\right)\right.\,.$$

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- Torsion contains #8 dof:

$$T^{t}{}_{tr} = t_1(r)\,, \quad T^{r}{}_{tr} = t_2(r)\,, \quad T^{\vartheta}{}_{t\vartheta} = T^{\varphi}{}_{t\varphi} = t_3(r)\,, \quad T^{\vartheta}{}_{r\vartheta} = T^{\varphi}{}_{r\varphi} = t_4(r)\,,$$
$$T^{\vartheta}{}_{t\varphi} = T^{\varphi}{}_{\vartheta t}\sin^2\vartheta = t_5(r)\sin\vartheta\,, \quad T^{\vartheta}{}_{r\varphi} = T^{\varphi}{}_{\vartheta r}\sin^2\vartheta = t_6(r)\sin\vartheta\,,$$
$$T^{t}{}_{\vartheta\varphi} = t_7(r)\sin\vartheta\,, \quad T^{r}{}_{\vartheta\varphi} = t_8(r)\sin\vartheta\,.$$

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- Nonmetricity contains $\#12$ dof:

$$Q_{ttt} = q_1(r)\,, \quad Q_{trr} = q_2(r)\,, \quad Q_{ttr} = q_3(r)\,,$$
$$Q_{t\vartheta\vartheta} = Q_{t\varphi\varphi}\csc^2\vartheta = q_4(r)\,, \quad Q_{rtt} = q_5(r)\,, \quad Q_{rrr} = q_6(r)\,,$$
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$$Q_{\vartheta t\vartheta} = Q_{\varphi t\varphi}\csc^2\vartheta = q_9(r)\,, \quad Q_{\vartheta r\vartheta} = Q_{\varphi r\varphi}\csc^2\vartheta = q_{10}(r)\,,$$
$$Q_{\vartheta t\varphi} = -Q_{\varphi t\vartheta} = q_{11}(r)\sin\vartheta\,, \quad Q_{\vartheta r\varphi} = -Q_{\varphi r\vartheta} = q_{12}(r)\sin\vartheta\,.$$

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- This means that not only the field equations are very difficult to treat but we need to find a solution of a system with $\#2(\text{metric})+\#8(\text{torsion})+\#12(\text{nonmetricity})=22$ dof!

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q_1(r) &= \frac{\Psi_1(r)}{r^2}\left(r^2 q_2(r)\Psi_2(r) + 2q_4(r)\right)\,, \\
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  2. We imposed $N_{[\lambda\rho]\mu} = 0$ which is equivalent to $T_{\lambda\mu\nu} = Q_{[\mu\nu]\lambda}$:
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- After following these three steps we end up with 2 dof (metric)+ 5 dof (torsion/nonmetricity) which is only 7 dof.

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Here, $\kappa_{\rm sh}$ is interpreted as a new charge, "shear charge".

- See our paper to see the form of q_i and t_i. One component of nonmetricity is arbitrary (problem?).

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- The action of the full model is

$$S = \frac{1}{64\pi}\int\Big[-4R - 6d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1\tilde{R}_{\lambda[\rho\mu\nu]}\tilde{R}^{\mu[\lambda\nu\rho]}$$
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$$- 3d_1\tilde{R}_{(\lambda\rho)\mu\nu}\tilde{R}^{(\lambda\rho)\mu\nu} + 6d_1\tilde{R}_{(\lambda\rho)\mu\nu}\tilde{R}^{(\lambda\mu)\rho\nu} + 2\left(2e_1 - f_1\right)\tilde{R}^{\lambda}{}_{\lambda\mu\nu}\tilde{R}^{\rho}{}_{\rho}{}^{\mu\nu}$$
$$+ 8f_1\tilde{R}_{(\lambda\rho)\mu\nu}\tilde{R}^{(\lambda\rho)\mu\nu} - 2f_1\left(\tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)}\right)\left(\tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)}\right)$$
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having the three possible charges of MAG: spin, dilation and shear.

On the other hand, the solution can also be trivially generalised to include the cosmological constant and Coulomb electromagnetic fields with electric and magnetic charges q_e and q_m, which are decoupled from torsion under the assumption of the minimal coupling principle. This natural extension is then described by a Reissner-Nordström-de Sitter-like geometry

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which turns out to represent **the broadest family of static and spherically symmetric black hole solutions obtained in MAG** so far.

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  4. We expect that the spin charge might be important in certain astrophysical scenarios such as: highly mangnetized neutron stars; supermassive black holes with endowed spin.

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- Do all particles in nature have different dilations and shears? are these properties important in particle physics?

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