

# Metric-Affine Gravity: from theory to applications in black holes and cosmology

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ICRAC2024, Pakistan, 01/Feb/2024

JCAP **09** (2020), 057+ JCAP **01** (2022) no.01, 011+ JCAP **04** (2022) no.04, 011+ JCAP **02**, 018 (2023)+Phys. Rev. D **108** (2023) no.4, 044037+arXiv:2310.16007

Jointly with **Jorge Gigante Valcarcel**, L. Järv, J. Chevrier, K.Aoki, M.A. Gorji.

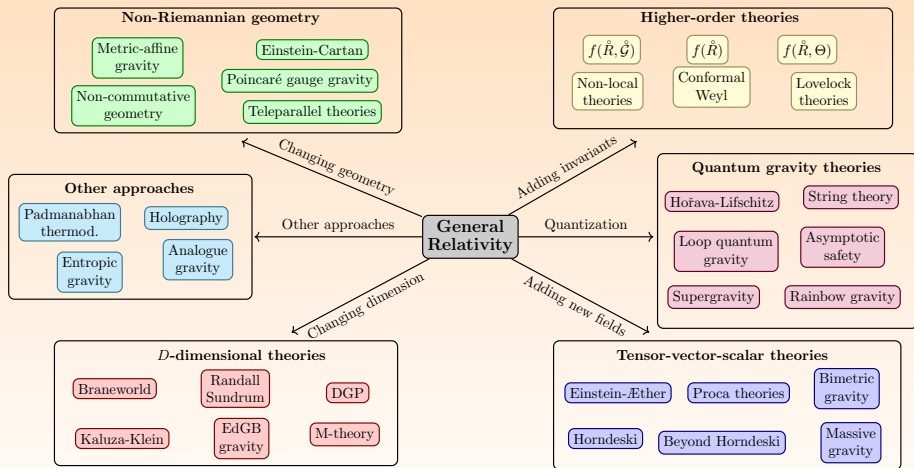
# Overview of the Talk

- 1 Introduction to Metric-affine gravity
- 2 Metric-Affine gravity
  - Basic geometrical quantities
- 3 MAG models with dynamical torsion and nonmetricity (Weyl only)
  - Spherically symmetry in Weyl-Cartan geometry
- 4 Cosmological perturbations in MAG

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# How to modify it?



**Figure:** Classification of theories of gravity. (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

# Post-Riemannian decomposition

- In the most general metric-affine setting, the fundamental variables are a **metric**  $g_{\mu\nu}$  (10 comp.) as well as the coefficients  $\tilde{\Gamma}^{\rho}_{\mu\nu}$  (64 comp.) of an **affine connection**.

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- Further, we can write

$$\begin{aligned}\tilde{\Gamma}^\lambda{}_{\mu\nu} &= \Gamma^\lambda{}_{\mu\nu} + K^\lambda{}_{\mu\nu} + L^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} + N^\lambda{}_{\mu\nu} \\ \tilde{R}^\lambda{}_{\rho\mu\nu} &= R^\lambda{}_{\rho\mu\nu} + 2\nabla_{[\mu} N^\lambda{}_{\rho|\nu]} + 2N^\lambda{}_{\sigma[\mu} N^\sigma{}_{\rho|\nu]}.\end{aligned}$$

# Decomposition into irreducible parts

- Irreducible decomposition of the torsion tensor:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3} \left( \delta^{\lambda}{}_{\nu} T_{\mu} - \delta^{\lambda}{}_{\mu} T_{\nu} \right) + \frac{1}{6} \varepsilon^{\lambda}{}_{\rho\mu\nu} S^{\rho} + t^{\lambda}{}_{\mu\nu} .$$

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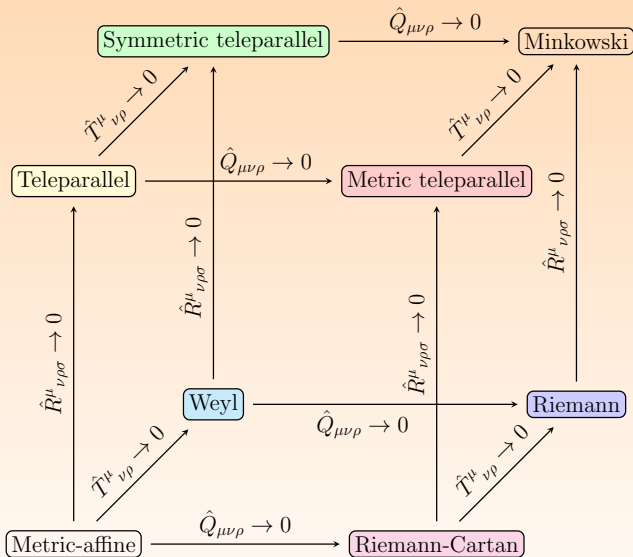
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**Figure: Classification of metric-affine geometries - Cube** (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

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- Quadratic gravitational action with dynamical torsion and nonmetricity in Weyl-Cartan geometry ( $Q_{\lambda\mu\nu} = g_{\mu\nu}W_\lambda$  and  $\tilde{Q}_{\lambda\mu\nu} = 0$ )

$$S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_m + \frac{1}{64\pi} \left[ -4R - 6d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{\mu[\lambda\nu\rho]} + 8d_1 \tilde{R}_{[\mu\nu]} \tilde{R}^{[\mu\nu]} + \frac{1}{8} (32e_1 + 8e_2 + 17d_1) \tilde{R}^\lambda{}_{\lambda\mu\nu} \tilde{R}^\rho{}_{\rho}{}^{\mu\nu} - 7d_1 \tilde{R}_{[\mu\nu]} \tilde{R}^\lambda{}_{\lambda}{}^{\mu\nu} + 3(1 - 2a_2) T_{[\lambda\mu\nu]} T^{[\lambda\mu\nu]} \right] \right\}.$$

<sup>1</sup> S. Bahamonde and J. G. Valcarcel, JCAP **09**, 057 (2020).

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- Absence of a general Birkhoff's theorem in MAG: new spherically and axially symmetric vacuum solutions with independent dynamical torsion and nonmetricity fields<sup>1,2</sup>

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# Spherical symmetry

- Metric, torsion and nonmetricity in spherically symmetric space-times (#2 + #8 + #2 = #12):

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- By solving these equations we find that torsion and nonmetricity behave as

$$\begin{aligned} T^t{}_{tr} &= a(r), & T^r{}_{tr} &= b(r), & T^{\theta_k}{}_{t\theta_k} &= f(r), & T^{\theta_k}{}_{r\theta_k} &= g(r) \\ T^{\theta_k}{}_{t\theta_l} &= e^{a\theta_k} e^b{}_{\theta_l} \epsilon_{ab} d(r), & T^{\theta_k}{}_{r\theta_l} &= e^{a\theta_k} e^b{}_{\theta_l} \epsilon_{ab} h(r), \\ T^t{}_{\theta_k\theta_l} &= \epsilon_{kl} k(r) \sin \theta_1, & T^r{}_{\theta_k\theta_l} &= \epsilon_{kl} l(r) \sin \theta_1, \\ W_\lambda &= (w_1(r), w_2(r), 0, 0), \end{aligned}$$

whereas the metric is in the standard spherically symmetric form:

$$ds^2 = \Psi_1(r) dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 (d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2).$$

Here,  $\epsilon_{kl}$  is the Levi-Civita symbol in two dimensions.

- Reissner-Nordström solution with spin and dilation charges:

$$g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4e_1 \kappa_d^2}{r^2} .$$

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- Ghost-free conditions for torsion and nonmetricity:

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- Absence of the inner pathological Cauchy horizon in the Reissner-Nordström geometry if

$$d_1 \kappa_s^2 - 4e_1 \kappa_{d,e}^2 \leq 0, \quad \Psi(r) = 1 - \frac{2m}{r} - \frac{|d_1 \kappa_s^2 - 4e_1 \kappa_{d,e}^2|}{r^2}.$$

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- 4 We expect that the spin charge might be important in certain astrophysical scenarios such as: highly magnetized neutron stars; supermassive black holes with endowed spin.

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- ③ Weyl part of nonmetricity is "scale invariant"

# What do these charges represent?

## ● **Nonmetricity part - only Weyl:**

- 1 Intrinsic dilations generates gravitation. This effect does not exist in GR.
- 2 dilation: deformation that involves only change of volume (in this case, intrinsic dilation!)
- 3 Weyl part of nonmetricity is "scale invariant"
- 4 Do all particles in nature have different dilations? is this property important in particle physics?

# The traceless part of nonmetricity and shears

- Recall that the complete nonmetricity is  $Q_{\lambda\mu\nu} = g_{\mu\nu}W_\lambda + \mathcal{Q}_{\lambda\mu\nu}$ , where  $\mathcal{Q}_{\lambda\mu\nu}$  being its traceless part.

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- If this quantity is different to zero, when we parallel transport a vector, not only its norm changes but also its angle.
- It is invariant under shears transformations.
- Shears: Deformations without changing the volume.
- Up to now, there were not exact solutions with shears in MAG.

# MAG theory with shears

- Let us first consider a simple model where torsion is not propagating and the traceless part of nonmetricity is dynamical:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ -R + 2f_1 \tilde{R}_{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} + 2f_2 \left( \tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)} \right) \left( \tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)} \right) \right],$$

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- As can be seen, the propagation of the nonmetricity field described in the action is carried out by the symmetric part of the curvature tensor and its symmetric contraction:

$$\begin{aligned} \tilde{R}^{(\lambda\rho)}_{\mu\nu} &= \tilde{\nabla}_{[\nu} Q_{\mu]}^{\lambda\rho} + \frac{1}{2} T^{\sigma}_{\mu\nu} Q_{\sigma}^{\lambda\rho}, \\ \tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)} &= \tilde{\nabla}_{(\mu} Q^{\lambda}_{\nu)\lambda} - \tilde{\nabla}_{\lambda} Q_{(\mu\nu)}^{\lambda} - Q^{\lambda\rho}_{\lambda} Q_{(\mu\nu)\rho} + Q_{\lambda\rho(\mu} Q_{\nu)}^{\lambda\rho} \\ &\quad + T_{\lambda\rho(\mu} Q^{\lambda\rho}_{\nu)}, \end{aligned}$$

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- Metric, torsion and nonmetricity in spherically symmetric space-times (#2 + #8 + #12 = #22):

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\alpha\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}_{\lambda\rho\mu\nu} = 0$$

# Spherical symmetry with nonmetricity and torsion

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- Nonmetricity now contains all the 12 dof:

$$\begin{aligned} Q_{ttt} &= q_1(r), & Q_{trr} &= q_2(r), & Q_{ttr} &= q_3(r), \\ Q_{t\vartheta\vartheta} &= Q_{t\varphi\varphi} \csc^2 \vartheta = q_4(r), & Q_{rtt} &= q_5(r), & Q_{rrr} &= q_6(r), \\ Q_{rtr} &= q_7(r), & Q_{r\vartheta\vartheta} &= Q_{r\varphi\varphi} \csc^2 \vartheta = q_8(r), \\ Q_{\vartheta t\vartheta} &= Q_{\varphi t\varphi} \csc^2 \vartheta = q_9(r), & Q_{\vartheta r\vartheta} &= Q_{\varphi r\varphi} \csc^2 \vartheta = q_{10}(r), \\ Q_{\vartheta t\varphi} &= -Q_{\varphi t\vartheta} = q_{11}(r) \sin \vartheta, & Q_{\vartheta r\varphi} &= -Q_{\varphi r\vartheta} = q_{12}(r) \sin \vartheta, \end{aligned}$$

whereas the metric and torsion are the same as before.

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$$q_1(r) = \frac{\Psi_1(r)}{r^2} (r^2 q_2(r) \Psi_2(r) + 2q_4(r)) ,$$

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- ② We imposed  $N_{[\lambda\rho]\mu} = 0$  which is equivalent to  $T_{\lambda\mu\nu} = Q_{[\mu\nu]\lambda}$ :  
→ Shear transformations in the general linear group involves the part of the anholonomic connection describing a shear current or charge to take values in the **symmetric traceless part** of the Lie algebra.

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- ③ We demand the corresponding torsion and nonmetricity scalars of the solution to be regular.
- After following these three steps we end up with 2 dof (metric)+ 5 dof (torsion/nonmetricity) which is only 7 dof.

# New solution only with shears

- By plugging these conditions in the field equations, there are several branches but only one has solutions with dynamical shears. This gives

$$g_{tt} = -1/g_{rr} \equiv \Psi(r) = 1 - \frac{2m}{r} - \frac{2f_1\kappa_{\text{sh}}^2}{r^2}, \text{ with } f_2 = -f_1/4.$$

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- The action of the full model is

$$\begin{aligned}
 S = \frac{1}{64\pi} \int & \left[ -4R - 6d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{\lambda[\rho\mu\nu]} - 9d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{\mu[\lambda\nu\rho]} \right. \\
 & + 2d_1 \left( \tilde{R}_{[\mu\nu]} + \hat{R}_{[\mu\nu]} \right) \left( \tilde{R}^{[\mu\nu]} + \hat{R}^{[\mu\nu]} \right) + 18d_1 \tilde{R}_{\lambda[\rho\mu\nu]} \tilde{R}^{(\lambda\rho)\mu\nu} \\
 & - 3d_1 \tilde{R}_{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} + 6d_1 \tilde{R}_{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\mu)\rho\nu} + 2(2e_1 - f_1) \tilde{R}^\lambda{}_{\lambda\mu\nu} \tilde{R}^\rho{}_{\rho}{}^{\mu\nu} \\
 & + 8f_1 \tilde{R}_{(\lambda\rho)\mu\nu} \tilde{R}^{(\lambda\rho)\mu\nu} - 2f_1 \left( \tilde{R}_{(\mu\nu)} - \hat{R}_{(\mu\nu)} \right) \left( \tilde{R}^{(\mu\nu)} - \hat{R}^{(\mu\nu)} \right) \\
 & \left. + 3(1 - 2a_2) T_{[\lambda\mu\nu]} T^{[\lambda\mu\nu]} \right] d^4x \sqrt{-g}.
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- When traceless part of nonmetricity is zero, the above action coincides with the first one.

# Complete solution of the full model

- One can combine the two solutions for the complete model and also trivially incorporate a cosmological constant and Coulomb electromagnetic fields with electric and magnetic charges  $q_e$  and  $q_m$ , which are decoupled from torsion under the assumption of the minimal coupling principle. This natural extension is then described by a Reissner-Nordström-de Sitter-like geometry

$$\Psi(r) = 1 - \frac{2m}{r} + \frac{d_1 \kappa_s^2 - 4e_1 \kappa_d^2 - 2f_1 \kappa_{sh}^2 + q_e^2 + q_m^2}{r^2} + \frac{\Lambda}{3} r^2,$$

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- The solution has the 3 possible charges in MAG: spin  $\kappa_s$ , dilation  $\kappa_d$  and shear  $\kappa_{sh}$ .

# Overview of the Talk

- 1 Introduction to Metric-affine gravity
- 2 Metric-Affine gravity
  - Basic geometrical quantities
- 3 MAG models with dynamical torsion and nonmetricity (Weyl only)
  - Spherically symmetry in Weyl-Cartan geometry
- 4 Cosmological perturbations in MAG

## 3 + 1 decomposition - settling notation

- Let us first introduce a projector tensor

$$P_{\mu\nu} \equiv g_{\mu\nu} + n_{\mu}n_{\nu} ,$$

which is orthogonal ( $P_{\mu\nu}n^{\mu} = 0$ ) to a timelike vector  $n_{\mu}$  satisfying  $n_{\mu}n^{\mu} = -1$ .

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- Let us introduce arrows on top of quantities to denote traceless-spatial and symmetric quantities, i.e.:

$$\vec{X}_{\mu_1 \dots \mu_n} \equiv P_{\mu_1}^{\alpha_1} \dots P_{\mu_n}^{\alpha_n} X_{\alpha_1 \dots \alpha_n},$$

$$\vec{X}_{\mu_1 \dots \mu_n} = \vec{X}_{(\mu_1 \dots \mu_n)},$$

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- One can then straightforwardly decompose scalar and vectors:

$$\phi = \phi, \quad X_\mu = \vec{X}_\mu - n_\mu X_*.$$

# Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0.$$

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}\bar{g} &= -\bar{n}_\mu \bar{n}_\nu dx^\mu \otimes dx^\nu + \bar{P}_{\mu\nu} dx^\mu \otimes dx^\nu = -N^2 dt \otimes dt + a^2 \gamma_{ij} dx^i \otimes dx^j, \\ \bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t) \bar{n}_{[\mu} \bar{P}_{\nu]}{}^\lambda + 2T_2(t) \bar{\varepsilon}^\lambda{}_{\mu\nu\rho} \bar{n}^\rho, \\ \bar{Q}^\lambda{}_{\mu\nu} &= 2Q_1(t) \bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t) \bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t) \bar{P}_{\lambda(\mu} \bar{n}_{\nu)},\end{aligned}$$

where  $\gamma_{ij} dx^i \otimes dx^j = \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2$ . Note that there are 5 independent functions coming from Post-Riemannian.

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where  $\gamma_{ij} dx^i \otimes dx^j = \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2$ . Note that there are 5 independent functions coming from Post-Riemannian.

- Our aim is to perform a 3 + 1+SVT-decomposition for all the tensors in MAG up to linear perturbations to then write:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad \bar{T}^\lambda{}_{\mu\nu} = \bar{T}^\lambda{}_{\mu\nu} + \delta T^\lambda{}_{\mu\nu}, \quad \bar{Q}^\lambda{}_{\mu\nu} = \bar{Q}^\lambda{}_{\mu\nu} + \delta Q^\lambda{}_{\mu\nu}.$$

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- The majority of these studies predominantly focus on torsion.
- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

## 3 + 1 decomposition for FLRW - metric sector

- A symmetric rank-2 tensor (as the metric) can be easily decomposed as 1 tensor, 1 vector and 2 scalars:

$$X_{(\mu\nu)} = n_\mu n_\nu X_{**} + \frac{1}{3} P_{\mu\nu} X_{***} + \vec{X}_{\mu\nu} - 2n_{(\mu} \vec{X}_{*\nu)}.$$

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- Then, the metric perturbation can be decomposed as

$$\delta g_{\mu\nu} = -\bar{n}_\mu \bar{n}_\nu \alpha + \bar{P}_{\mu\nu} \zeta + \vec{h}_{\mu\nu} - 2\bar{n}_{(\mu} \vec{\beta}_{\nu)},$$

in such a way that the full metric tensor is then given by

$$\bar{g}_{\mu\nu} + \delta g_{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} -N^2(1 + \alpha) & aN\vec{\beta}_i \\ aN\vec{\beta}_j & a^2[(1 + \zeta)\gamma_{ij} + \vec{h}_{ij}] \end{pmatrix}.$$

## 3 + 1 decomposition for FLRW - Torsion sector

- The irreducible modes of antisymmetric rank-3 tensor (as torsion) can be decomposed as

$$T_\mu = \vec{T}_\mu - n_\mu \phi,$$

$$S_\mu = \vec{S}_\mu - n_\mu \varrho,$$

$$t_{\mu\nu\rho} = 2n_{[\nu}\vec{A}_{\rho]\mu} + 2\left(n_\mu n_{[\nu} + \frac{1}{2}P_{\mu[\nu}\right)\vec{B}_{\rho]} + \frac{1}{2}\varepsilon_{\nu\rho}{}^{\alpha\beta}\left[-n_\alpha\vec{A}_{\beta\mu} + \left(n_\beta n_\mu + \frac{1}{2}P_{\beta\mu}\right)\vec{B}_\alpha\right].$$

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$$t_{\mu\nu\rho} = 2n_{[\nu}\vec{A}_{\rho]\mu} + 2\left(n_\mu n_{[\nu} + \frac{1}{2}P_{\mu[\nu}\right)\vec{B}_{\rho]} + \frac{1}{2}\varepsilon_{\nu\rho}{}^{\alpha\beta}\left[-n_\alpha\vec{A}_{\beta\mu} + \left(n_\beta n_\mu + \frac{1}{2}P_{\beta\mu}\right)\vec{B}_\alpha\right].$$

- Then, torsion is decomposed as

$$T^\lambda{}_{\mu\nu} = \bar{T}^\lambda{}_{\mu\nu} + \delta T^\lambda{}_{\mu\nu} = \begin{cases} T^0{}_{0i} = -\frac{a}{3}(\vec{T}_i + 3\vec{B}_i) \\ T^0{}_{ij} = -\frac{a^2}{6N}\varepsilon_{ijk}(\vec{S}^k + 3\vec{B}^k) \\ T^i{}_{0j} = -N\left[\vec{A}^i{}_j - \frac{1}{3}\delta^i{}_j(\bar{\phi} + \phi) + \frac{1}{6}\varepsilon^i{}_{jk}(\vec{S}^k - \frac{3}{2}\vec{B}^k)\right] \\ T^i{}_{jk} = -\frac{a}{6}\left\{4\delta^i{}_{[j}\vec{T}_{k]} - 6\delta^i{}_{[j}\vec{B}_{k]} + \left[3\varepsilon_{jkl}\vec{A}^{il} - \varepsilon^i{}_{jk}(\bar{\varrho} + \varrho)\right]\right\} \end{cases}$$

where we have introduced the background  $T_i$  in the scalars  $\phi, \varrho$ .

## 3 + 1 decomposition for FLRW - Nonmetricity sector

- The irreducible modes of symmetric rank-3 tensor (as nonmetricity) can be decomposed as

$$W_\mu = \vec{W}_\mu - n_\mu \theta,$$

$$\Lambda_\mu = \vec{\Lambda}_\mu - n_\mu \psi,$$

$$\Omega_{\mu\nu\rho} = 2n_{[\nu} \vec{Q}_{\rho]\mu} + 2\left(n_\mu n_{[\nu} + \frac{1}{2}P_{\mu[\nu}\right) \vec{Y}_{\rho]} + \frac{1}{2}\varepsilon_{\nu\rho}{}^{\alpha\beta} \left[-n_\alpha \vec{Q}_{\beta\mu} + \left(n_\beta n_\mu + \frac{1}{2}P_{\beta\mu}\right) \vec{Y}_\alpha\right],$$

$$q_{\mu\nu\rho} = \vec{C}_{\mu\nu\rho} - 3n_{(\mu} \vec{\kappa}_{\nu\rho)} + \frac{3}{5}(5n_{(\mu} n_\nu + P_{(\mu\nu)}) \vec{Z}_{\rho)} - (n_{(\mu} n_\nu + P_{(\mu\nu)}) n_\rho) \xi.$$

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- Then, nonmetricity is decomposed as

$$Q^\lambda{}_{\mu\nu} = \begin{cases} Q_{000} = -N^3 \left[ \bar{\theta} + \theta + \frac{3}{4}(\bar{\psi} + \psi) - (\bar{\xi} + \xi) \right] \\ Q_{00i} = -\frac{N^2 a}{2} \left( \vec{\Lambda}_i - \frac{1}{3} \vec{Y}_i - 2\vec{Z}_i \right) \\ Q_{0ij} = -\frac{Na^2}{3} \left\{ \vec{Q}_{ij} - 3\vec{\kappa}_{ij} - 3 \left[ \bar{\theta} + \theta - \frac{1}{4}(\bar{\psi} + \psi) + \frac{1}{3}(\bar{\xi} + \xi) \right] \gamma_{ij} \right\} \\ Q_{i00} = -N^2 a \left( \vec{W}_i - \frac{1}{4} \vec{\Lambda}_i + \frac{1}{3} \vec{Y}_i - \vec{Z}_i \right) \\ Q_{i0j} = \frac{Na^2}{6} \left\{ [\vec{Q}_{ij} + 6\vec{\kappa}_{ij} + (3(\bar{\psi} + \psi) + 2(\bar{\xi} + \xi)) \gamma_{ij}] - 3\varepsilon_{ijk} \vec{Y}^k \right\} \\ Q_{ijk} = a^3 \left\{ \vec{C}_{ijk} + \frac{1}{4}(4\vec{W}_i - \vec{\Lambda}_i) \gamma_{jk} + \gamma_{i(j} \vec{\Lambda}_{k)} - \frac{1}{6}(\gamma_{jk} \vec{Y}_i - \gamma_{i(j} \vec{Y}_{k)}) + \frac{3}{5} \gamma_{(ij} \vec{Z}_{k)} + \frac{2}{3} \varepsilon_{li(j} \vec{Q}^l{}_{k)} \right\} \end{cases}$$

where we have introduced the background  $Q_i$  in the scalars  $\theta, \psi, \xi$ .

# Summary of modes

- We can group the modes as:

$$\delta\vec{\mathbf{X}} = \{\alpha, \zeta, \phi, \varrho, \theta, \psi, \xi\},$$

$$\delta\vec{\mathbf{X}}_i = \{\vec{\beta}_i, \vec{T}_i, \vec{S}_i, \vec{B}_i, \vec{\mathcal{B}}_i, \vec{W}_i, \vec{\Lambda}_i, \vec{Y}_i, \vec{\mathcal{Y}}_i, \vec{Z}_i\},$$

$$\delta\vec{\mathbf{X}}_{ij} = \{\vec{h}_{ij}, \vec{A}_{ij}, \vec{\mathcal{A}}_{ij}, \vec{Q}_{ij}, \vec{\mathcal{Q}}_{ij}, \vec{\kappa}_{ij}\},$$

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Spin and Parity	0 <sup>+</sup>	0 <sup>-</sup>	1 <sup>+</sup>	1 <sup>-</sup>	2 <sup>+</sup>	2 <sup>-</sup>	3 <sup>-</sup>
<b>Metric sector</b> $g_{\mu\nu}$	$\alpha, \zeta$	-	-	$\vec{\beta}_i$	$\vec{h}_{ij}$	-	-
<b>Torsion sector</b> $T^\lambda{}_{\mu\nu}$	$\phi$	$\varrho$	$\vec{S}_i, \vec{\mathcal{B}}_i$	$\vec{T}_i, \vec{B}_i$	$\vec{A}_{ij}$	$\vec{\mathcal{A}}_{ij}$	-
<b>Nonmetricity sector</b> $Q_{\lambda\mu\nu}$	$\theta, \psi, \xi$	-	$\vec{\mathcal{Y}}_i$	$\vec{W}_i, \vec{\Lambda}_i, \vec{Y}_i, \vec{Z}_i$	$\vec{Q}_{ij}, \vec{\kappa}_{ij}$	$\vec{\mathcal{Q}}_{ij}$	$\vec{C}_{ijk}$

**Table:** Species in MAG. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," [arXiv:2310.16007 [gr-qc]].

# Helicity (SVT) decomposition

- After the  $3 + 1$  decomposition, we only need to deal with the spatial scalar and tensors with Latin indices belonging to the maximally symmetric space  $\gamma_{ij}$  (and with the “external” parameter  $t$ ).

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- After the 3 + 1 decomposition, we only need to deal with the spatial scalar and tensors with Latin indices belonging to the maximally symmetric space  $\gamma_{ij}$  (and with the “external” parameter  $t$ ).
- Let us decompose them into different helicity sectors:

$$\delta \vec{\mathbf{X}}_i = D_i \mathbf{S} + \mathbf{V}_i^{(T)},$$

$$\delta \vec{\mathbf{X}}_{ij} = \left( D_{(i} D_{j)} - \frac{1}{3} \gamma_{ij} D^2 \right) \mathbf{S} + D_{(i} \mathbf{V}_{j)}^{(T)} + \mathbf{T}_{ij}^{(TT)},$$

$$\delta \vec{\mathbf{X}}_{ijk} = \left[ D_{(i} D_j D_{k)} - \frac{1}{5} \gamma_{(ij} D_{k)} (3D^2 + 4K) \right] \mathbf{S} + \left[ D_{(i} D_{j)} - \frac{1}{5} \gamma_{(ij} (D^2 + 2K) \right] \mathbf{V}_{k)}^{(T)} + D_{(i} \mathbf{T}_{jk)}^{(TT)}$$

Here, the superscript “(T)” refers that the quantity is transverse, while the tensors denoted by the superscript “(TT)” are transverse-traceless and symmetric:

$$D^i \mathbf{V}_i^{(T)} = 0,$$

$$D^i \mathbf{T}_{ij}^{(TT)} = \mathbf{T}_{ij}^{(TT)} \gamma^{ij} = 0, \quad \mathbf{T}_{ij}^{(TT)} = \mathbf{T}_{(ij)}^{(TT)},$$

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# SVT decomposition around FRW

- Thereby, the 10 dof described by the metric perturbations are split in terms of four scalars  $\{\alpha, \beta, \zeta, h\}$  (1 dof each), two transverse vectors  $\{\beta_i^{(T)}, h_i^{(T)}\}$  (2 dof each), and one symmetric and transverse-traceless tensor  $h_{ij}^{(TT)}$  (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(T)}, B_i^{(T)}, A_i^{(T)}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(T)}, \mathcal{B}_i^{(T)}, \mathcal{A}_i^{(T)}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(TT)}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(TT)}\}$	2 dof each	2

**Table:** Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," [arXiv:2310.16007 [gr-qc]].

# SVT decomposition around FRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\text{T})}_i, Y^{(\text{T})}_i, Z^{(\text{T})}_i, \kappa^{(\text{T})}_i, Q^{(\text{T})}_i, W^{(\text{T})}_i, C^{(\text{T})}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(\text{T})}_i, \mathcal{Q}^{(\text{T})}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(\text{TT})}_{ij}, Q^{(\text{TT})}_{ij}, C^{(\text{TT})}_{ij}\}$	2 dof each	6
1 rank-2 pseudotensor	$\{\mathcal{Q}^{(\text{TT})}_{ij}\}$	2 dof each	2
1 rank-3 tensor	$\{C^{(\text{TTT})}_{ijk}\}$	2 dof each	2

**Table:** Perturbation spectrum for the nonmetricity tensor.

# SVT decomposition around FRW

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2 pseudovectors	$\{\mathcal{Y}^{(T)}_i, \mathcal{Q}^{(T)}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(TT)}_{ij}, Q^{(TT)}_{ij}, C^{(TT)}_{ij}\}$	2 dof each	6
1 rank-2 pseudotensor	$\{\mathcal{Q}^{(TT)}_{ij}\}$	2 dof each	2
1 rank-3 tensor	$\{C^{(TTT)}_{ijk}\}$	2 dof each	2

**Table:** Perturbation spectrum for the nonmetricity tensor.

Note that the spin-3 carries a helicity-3 part and since the metric does not have such terms, it totally decouples from the other modes in FLRW.

# Dynamics of metric-affine geometry

- General quadratic parity preserving action with dynamical torsion and nonmetricity:

$$\begin{aligned} S = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_m + \frac{1}{16\pi} \left[ -\tilde{R} + a_1 \tilde{R}^2 + a_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} + a_3 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\rho\lambda\mu\nu} \right. \right. \\ + a_4 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\lambda\rho} + a_5 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} + a_6 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\lambda\rho\nu} + a_7 \tilde{R}_{\rho\lambda\mu\nu} \tilde{R}^{\mu\lambda\rho\nu} \\ + a_8 \tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu} + a_9 \tilde{R}_{\mu\nu} \tilde{R}^{\nu\mu} + a_{10} \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + a_{11} \hat{R}_{\mu\nu} \hat{R}^{\nu\mu} + a_{12} \tilde{R}_{\mu\nu} \hat{R}^{\mu\nu} \\ + a_{13} \tilde{R}_{\mu\nu} \hat{R}^{\nu\mu} + a_{14} \tilde{R}^\lambda{}_{\lambda\mu\nu} \tilde{R}^\rho{}_{\rho}{}^{\mu\nu} + a_{15} \tilde{R}^\lambda{}_{\lambda\mu\nu} \tilde{R}^{\mu\nu} + a_{16} \tilde{R}^\lambda{}_{\lambda\mu\nu} \hat{R}^{\mu\nu} \\ + b_1 T_{\lambda\mu\nu} T^{\lambda\mu\nu} + b_2 T_{\lambda\mu\nu} T^{\mu\lambda\nu} + b_3 T^\lambda{}_{\lambda\nu} T^\mu{}_{\mu}{}^\nu + c_1 T_{\lambda\mu\nu} Q^{\mu\lambda\nu} \\ + c_2 T^\lambda{}_{\lambda\nu} Q^{\nu\mu}{}_{\mu} + c_3 T^\lambda{}_{\lambda\nu} Q^{\mu\nu}{}_{\mu} + d_1 Q_{\lambda\mu\nu} Q^{\lambda\mu\nu} + d_2 Q_{\lambda\mu\nu} Q^{\mu\lambda\nu} \\ \left. + d_3 Q^\lambda{}_{\lambda\nu} Q^\mu{}_{\mu}{}^\nu + d_4 Q_\nu{}^\lambda{}_\lambda Q^{\nu\mu}{}_{\mu} + d_5 Q^\lambda{}_{\lambda\nu} Q^{\nu\mu}{}_{\mu} \right] \}. \end{aligned}$$

- Contribution of the spin-3 field (helicity-3 part) around flat FRW up to second order for the most general parity-preserving quadratic MAG action:

$$\begin{aligned}
 S_C^{(2)} &= \frac{1}{16\pi} \int dt d^3x a^3 \mathcal{G}_T^{(C)} \left[ \left( \dot{C}_{ijk}^{(TT)} \right)^2 - \frac{1}{a^2} \left( D_l C_{ijk}^{(TT)} \right)^2 + \frac{4}{a} T_2 \varepsilon_{ijk} C^{(TT) i}{}_{lm} D^j C^{(TT) klm} \right. \\
 &\quad \left. - m_{C,\text{eff}}^2 \left( C_{ijk}^{(TT)} \right)^2 \right] \\
 &= \frac{1}{16\pi} \sum_{A=L,R} \int dt \frac{d^3k}{(2\pi)^3} a^3 \mathcal{G}_T^{(C)} \left[ [\dot{C}^{(A)}]^2 - [\omega_{C,k}^{(A)}]^2 [C^{(A)}]^2 \right],
 \end{aligned}$$

where  $[\omega_{C,k}^{(A)}]^2 \equiv \frac{k^2}{a^2} - 4\lambda_A T_2 \frac{k}{a} + m_{C,\text{eff}}^2$  and

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- Field equation of the spin-3 field around flat FRW:

$$\ddot{C}^{(A)} + 3H\dot{C}^{(A)} + [\omega_{C,k}^{(A)}]^2 C^{(A)} = J^{(A)}.$$

- The quadratic action contains the parity-violating term

$$T_2 \varepsilon_{ijk} C^{(\text{TT})i}{}_{lm} D^j C^{(\text{TT})klm} \rightarrow \lambda_A T_2 k [C^{(A)}]^2 \quad (\text{momentum space}),$$

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- Another direction is to study cosmology: there are many new fields in MAG and the cosmological perturbation theory was remained unexplored in the literature.
- We have derived all the possible cosmological perturbations arising from MAG, by performing 3+1 and SVT decompositions over the metric, torsion and nonmetricity tensors.

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- What is the role of dilations/shears in particle physics?
- Some possible new quantum gravity theories with torsion and nonmetricity?