

Stability in Cubic Metric-Affine Gravity

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Jointly with Jorge Gigante Valcarcel.



- 1 Motivations and introduction to metric-affine geometry
- 2 Stability in cubic Poincaré gauge theory
 - Stability issues in quadratic Poincaré gauge theory
 - Cubic extensions
- 3 Metric-Affine gravity and stability
- 4 Conclusions and future prospects

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- Metric and connection are independent quantities and the new dof are known as torsion and nonmetricity

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$$Q_{\lambda\mu\nu} = \tilde{\nabla}_\lambda g_{\mu\nu} \quad 40 \text{ dof} \quad (\text{Measures the change of lengths and angles under parallel transport})$$

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- It is possible to perform a post-Riemannian expansion using

$$\begin{aligned} \tilde{\Gamma}^\lambda{}_{\mu\nu} &= \Gamma^\lambda{}_{\mu\nu} + N^\lambda{}_{\mu\nu}, \\ N^\lambda{}_{\mu\nu} &= \frac{1}{2} T^\lambda{}_{\mu\nu} - T_{(\mu}{}^\lambda{}_{\nu)} + \frac{1}{2} Q^\lambda{}_{\mu\nu} - Q_{(\mu}{}^\lambda{}_{\nu)}. \end{aligned}$$

Symmetries and identities of the curvature tensor

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- Three independent rank-2 tensors defined from the first contractions of the curvature tensor:

$$\tilde{R}_{\mu\nu} = \tilde{R}^{\lambda}{}_{\mu\lambda\nu}, \quad \hat{R}_{\mu\nu} = \tilde{R}_{\mu}{}^{\lambda}{}_{\nu\lambda}, \quad \tilde{R}^{\lambda}{}_{\lambda\mu\nu} = \nabla_{[\nu} Q_{\mu]}{}^{\lambda}{}_{\lambda}.$$

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- Unique scalar and pseudoscalar curvatures:

$$\tilde{R} = g^{\mu\nu} \tilde{R}_{\mu\nu}, \quad * \tilde{R} = \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu}.$$

Decomposition into irreducible parts

Irreducible decomposition of the torsion tensor

$$T^\lambda{}_{\mu\nu} = \frac{1}{3} (\delta^\lambda{}_\nu T_\mu - \delta^\lambda{}_\mu T_\nu) + \frac{1}{6} \varepsilon^\lambda{}_{\rho\mu\nu} S^\rho + t^\lambda{}_{\mu\nu}.$$

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Irreducible decomposition of the nonmetricity tensor

$$Q_{\lambda\mu\nu} = g_{\mu\nu} W_\lambda + \mathcal{Q}_{\lambda\mu\nu},$$

$$\mathcal{Q}_{\lambda\mu\nu} = \frac{1}{2} (g_{\lambda\mu} \Lambda_\nu + g_{\lambda\nu} \Lambda_\mu) - \frac{1}{4} g_{\mu\nu} \Lambda_\lambda + \frac{1}{3} \varepsilon_{\lambda\rho\sigma(\mu} \Omega_{\nu)}{}^{\rho\sigma} + q_{\lambda\mu\nu}.$$

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- Second tensor part $q_{\lambda\mu\nu} = Q_{(\lambda\mu\nu)} - g_{(\mu\nu} W_{\lambda)} - \frac{3}{4} g_{(\mu\nu} \Lambda_{\lambda)}$.

- Class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion:

$$S_g = \frac{1}{16\pi} \int \left[c_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} - R - \frac{1}{2} (2c_1 + c_2) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\lambda\rho} + c_1 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} + d_1 \tilde{R}_{\mu\nu} (\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}) + \frac{1}{2} (m_T^2 T_\mu T^\mu + m_S^2 S_\mu S^\mu + m_t^2 t_{\lambda\mu\nu} t^{\lambda\mu\nu}) \right] \sqrt{-g} d^4x.$$

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- Focusing on the vector and axial modes of torsion:

$$\begin{aligned} \tilde{R}_{\lambda\rho\mu\nu} &= R_{\lambda\rho\mu\nu} + \frac{2}{3} (g_{\lambda[\nu} \nabla_{\mu]} T_{\rho]} - g_{\rho[\nu} \nabla_{\mu]} T_{\lambda]}) + \frac{1}{6} \varepsilon_{\lambda\rho\sigma[\mu} \nabla_{\nu]} S^\sigma \\ &+ \frac{2}{9} (2T_{[\lambda} g_{\rho][\nu} T_{\mu]} + g_{\lambda[\nu} g_{\mu]\rho} T_\sigma T^\sigma) + \frac{1}{72} (2S_{[\lambda} g_{\rho][\mu} S_{\nu]} + g_{\lambda[\mu} g_{\nu]\rho} S_\sigma S^\sigma) \\ &+ \frac{1}{18} [2\varepsilon_{\mu\nu\sigma[\lambda} T_{\rho]} S^\sigma + (g_{\lambda[\nu} \varepsilon_{\mu]\rho\sigma\omega} - g_{\rho[\nu} \varepsilon_{\mu]\lambda\sigma\omega}) T^\sigma S^\omega] , \\ \tilde{R}_{\mu\nu} &= R_{\mu\nu} - \frac{1}{3} (2\nabla_\nu T_\mu + g_{\mu\nu} \nabla_\lambda T^\lambda) + \frac{1}{12} \varepsilon_{\lambda\rho\mu\nu} \nabla^\lambda S^\rho \\ &+ \frac{2}{9} (T_\mu T_\nu - g_{\mu\nu} T_\lambda T^\lambda) + \frac{1}{72} (g_{\mu\nu} S_\lambda S^\lambda - S_\mu S_\nu) . \end{aligned}$$

- Instabilities in the vector and axial sectors:

$$\begin{aligned}\mathcal{L}_g^{(2)} \propto & \frac{1}{9} (4c_1 + c_2 + 2d_1) F_{\mu\nu}^{(T)} F^{(T)\mu\nu} - \frac{1}{72} (4c_1 + c_2 + d_1) F_{\mu\nu}^{(S)} F^{(S)\mu\nu} \\ & + \frac{1}{24} (c_2 - 2c_1) \nabla_\mu S^\mu \nabla_\nu S^\nu + \frac{1}{27} (c_1 - 2c_2) S^\mu S^\nu \nabla_\mu T_\nu \\ & - \frac{1}{108} (4c_1 + c_2) S_\mu S^\mu \nabla_\nu T^\nu + \frac{1}{18} (2c_1 - c_2) T^\mu S^\nu \nabla_\nu S_\mu \\ & + \frac{1}{36} (4c_1 + c_2) G_{\mu\nu} S^\mu S^\nu + \frac{1}{72} (4c_1 + c_2) R S_\mu S^\mu \\ & - \frac{1}{162} (4c_1 + c_2) T_\mu T^\mu S_\nu S^\nu + \frac{1}{81} (2c_2 - c_1) T_\mu S^\mu T_\nu S^\nu \\ & + \frac{1}{2} m_T^2 T_\mu T^\mu + \frac{1}{2} m_S^2 S_\mu S^\mu .\end{aligned}$$

and then $c_1 = c_2 = d_1 = 0$ (only the masses survives in general backgrounds!)

Vector and axial sectors in quadratic Poincaré gauge theory

- Instabilities in the vector and axial sectors:

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and then $c_1 = c_2 = d_1 = 0$ (only the masses survives in general backgrounds!)

Important problem!

It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms:¹

$$\mathcal{L}_{\text{curv-tors}}^{(3)} = \mathcal{L}_{\tilde{R}TT}^{(3)} + \mathcal{L}_{\tilde{R}SS}^{(3)} + \mathcal{L}_{\tilde{R}tt}^{(3)} + \mathcal{L}_{\tilde{R}TS}^{(3)} + \mathcal{L}_{\tilde{R}Tt}^{(3)} + \mathcal{L}_{\tilde{R}St}^{(3)},$$

$$\mathcal{L}_{\tilde{R}TT}^{(3)} = h_1 \tilde{R}_{\mu\nu} T^\mu T^\nu + h_2 \tilde{R} T_\mu T^\mu, \quad \mathcal{L}_{\tilde{R}SS}^{(3)} = h_3 \tilde{R}_{\mu\nu} S^\mu S^\nu + h_4 \tilde{R} S_\mu S^\mu,$$

$$\begin{aligned} \mathcal{L}_{\tilde{R}tt}^{(3)} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho} t^{\sigma\mu\nu} \\ &+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}^{\lambda\rho} t^{\rho\mu\nu} \\ &+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}^{\lambda\rho} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho} t^{\lambda\rho\mu}, \end{aligned}$$

$$\mathcal{L}_{\tilde{R}TS}^{(3)} = h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_\sigma S^\sigma + h_{14} \varepsilon_\nu^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^\mu S^\nu + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_\mu S_\nu,$$

$$\mathcal{L}_{\tilde{R}Tt}^{(3)} = h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^\nu t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^\rho t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_\mu t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_\mu t^{\lambda\rho\mu},$$

$$\begin{aligned} \mathcal{L}_{\tilde{R}St}^{(3)} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau^{\rho\mu\nu} S^\gamma t^{\alpha\tau}_\gamma + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau^{\rho\mu\nu} S^\gamma t_\gamma^{\alpha\tau} \\ &+ h_{22} \varepsilon_{\alpha\rho}^{\mu\nu} \tilde{R}^\rho_{\mu\tau\nu} S^\gamma t_\gamma^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\gamma\rho\tau} \\ &+ h_{24} \varepsilon_{\alpha\rho}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^\mu_\gamma S^\alpha t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_\sigma t^{\sigma\mu\nu}. \end{aligned}$$

¹ S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10

- Stable vector and axial sectors:

$$\begin{aligned}
 \mathcal{L} \propto & -R + \frac{1}{9} (4c_1 + c_2 + 2d_1) F_{\mu\nu}^{(T)} F^{(T)\mu\nu} - \frac{1}{72} (4c_1 + c_2 + d_1) F_{\mu\nu}^{(S)} F^{(S)\mu\nu} \\
 & + \frac{1}{24} (c_2 - 2c_1) \nabla_\mu S^\mu \nabla_\nu S^\nu + \frac{1}{54} [2c_1 - 4c_2 - 9(4h_3 + 6h_{13} + h_{14})] S^\mu S^\nu \nabla_\mu T_\nu \\
 & - \frac{1}{108} [4c_1 + c_2 + 9(4h_3 + 24h_4 + 2h_{14} - h_{15})] S_\mu S^\mu \nabla_\nu T^\nu \\
 & + \frac{1}{18} [2c_1 - c_2 - 3(6h_{13} + h_{14} - h_{15})] T^\mu S^\nu \nabla_\nu S_\mu - 2h_2 T_\mu T^\mu \nabla_\nu T^\nu \\
 & - \frac{2}{3} (h_{14} - h_{15}) \varepsilon^{\lambda\rho\mu\nu} T_\lambda S_\rho \partial_\mu T_\nu + \frac{1}{36} (4c_1 + c_2 + 36h_3) G_{\mu\nu} S^\mu S^\nu \\
 & + \frac{1}{72} [4c_1 + c_2 + 36(h_3 + 2h_4)] RS_\mu S^\mu + h_1 G_{\mu\nu} T^\mu T^\nu + \frac{1}{2} (h_1 + 2h_2) RT_\mu T^\mu \\
 & - \frac{1}{648} [16c_1 + 4c_2 - 9(h_1 + 3h_2 - 16h_3 - 48h_4 - 8h_{14})] T_\mu T^\mu S_\nu S^\nu \\
 & + \frac{1}{648} [16c_2 - 8c_1 - 9(h_1 - 16h_3 - 48h_{13} - 8h_{14})] T_\mu S^\mu T_\nu S^\nu \\
 & - \frac{2}{3} h_2 T_\mu T^\mu T_\nu T^\nu + \frac{1}{24} h_4 S_\mu S^\mu S_\nu S^\nu + \frac{1}{2} m_T^2 T_\mu T^\mu + \frac{1}{2} m_S^2 S_\mu S^\mu .
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- First stability constraints (6 equalities):

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$$\begin{aligned} \mathcal{L} \propto & -R + \frac{2}{9}(3c_1 + d_1) F_{\mu\nu}^{(T)} F^{(T)\mu\nu} - \frac{1}{72}(6c_1 + d_1) F_{\mu\nu}^{(S)} F^{(S)\mu\nu} \\ & + 2h_1 T_\mu T^\mu \nabla_\nu T^\nu + 4h_{13} \varepsilon^{\lambda\rho\mu\nu} T_\lambda S_\rho \partial_\mu T_\nu + h_1 G_{\mu\nu} T^\mu T^\nu - h_{13} G_{\mu\nu} S^\mu S^\nu \\ & + \frac{1}{144}(16h_{13} - h_1) T_\mu T^\mu S_\nu S^\nu + \frac{1}{72}(16h_{13} - h_1) T_\mu S^\mu T_\nu S^\nu \\ & + \frac{1}{3} h_1 T_\mu T^\mu T_\nu T^\nu + \frac{1}{48} h_{13} S_\mu S^\mu S_\nu S^\nu + \frac{1}{2} m_T^2 T_\mu T^\mu + \frac{1}{2} m_S^2 S_\mu S^\mu. \end{aligned}$$

Complete theory contains 23+3 constants. Clearly $-\frac{d_1}{6} \leq c_1 \leq -\frac{d_1}{3}, d_1 \leq 0$.

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Conclusion 1: Problem solved

By adding Cubic interactions, it is possible to construct a stable theory with propagating torsion in the gauge formalism

Quadratic Metric-Affine Gravity

The most general quadratic gravitational action of MAG that is reduced to GR in the absence of torsion and nonmetricity is

$$\begin{aligned} S = \frac{1}{16\pi} \int & \left[-R - \frac{1}{2} (2c_1 + c_2) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\lambda\rho} + (a_2 - c_1) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\rho\lambda\mu\nu} + a_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} \right. \\ & + (c_2 - a_5 + a_6) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\rho\lambda\nu} + a_5 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} + a_6 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\rho\mu\lambda\nu} + (d_1 - a_{10} - a_{12}) \tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu} \\ & + a_9 \tilde{R}_{\mu\nu} \tilde{R}^{\nu\mu} + a_{10} \hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + a_{11} \hat{R}_{\mu\nu} \hat{R}^{\nu\mu} - (d_1 + a_9 + a_{11}) \tilde{R}_{\mu\nu} \hat{R}^{\nu\mu} + a_{12} \tilde{R}_{\mu\nu} \hat{R}^{\mu\nu} \\ & + a_{14} \tilde{R}^\lambda{}_{\lambda\mu\nu} \tilde{R}^\rho{}_{\rho}{}^{\mu\nu} + a_{15} \tilde{R}_{\mu\nu} \tilde{R}^\lambda{}_{\lambda}{}^{\mu\nu} + a_{16} \hat{R}_{\mu\nu} \hat{R}^\lambda{}_{\lambda}{}^{\mu\nu} \\ & + \frac{1}{2} m_T^2 T_\mu T^\mu + \frac{1}{2} m_S^2 S_\mu S^\mu + \frac{1}{2} m_t^2 t_{\lambda\mu\nu} t^{\lambda\mu\nu} + \frac{1}{2} m_W^2 W_\mu W^\mu + \frac{1}{2} m_\Lambda^2 \Lambda_\mu \Lambda^\mu + \frac{1}{2} m_\Omega^2 \Omega_{\lambda\mu\nu} \Omega^{\lambda\mu\nu} \\ & \left. + \frac{1}{2} m_q^2 q_{\lambda\mu\nu} q^{\lambda\mu\nu} + \frac{1}{2} \alpha_{TW} T_\mu W^\mu + \frac{1}{2} \alpha_{T\Lambda} T_\mu \Lambda^\mu + \frac{1}{2} \alpha_{W\Lambda} W_\mu \Lambda^\mu + \frac{1}{2} \alpha_{t\Omega} \varepsilon_{\rho\mu\nu\sigma} \Omega^{\lambda\rho\mu} t^\nu{}_{\lambda\sigma} \right] \sqrt{-g} d^4x. \end{aligned}$$

As can be seen, the action includes a total:

- 13 coefficients $c_1, c_2, d_1, a_2, a_5, a_6, a_9, a_{10}, a_{11}, a_{12}, a_{14}, a_{15}$ and a_{16} associated with the quadratic curvature invariants (only c_1, c_2, d_1 contribute in Poincare)

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- 11 coefficients $m_T, m_S, m_t, m_W, m_\Lambda, m_\Omega, m_q, \alpha_{TW}, \alpha_{T\Lambda}, \alpha_{W\Lambda}$ and $\alpha_{t\Omega}$ associated with the quadratic torsion and nonmetricity invariant.
All of them are stable (with the correct signs)

Quadratic Metric-Affine Gravity - Vector/Axial Stability

- Let us focus on the axial and vector sectors. Let us recall the decomposition:

$$T^\lambda{}_{\mu\nu} = \frac{1}{3} \left(\delta^\lambda{}_\nu T_\mu - \delta^\lambda{}_\mu T_\nu \right) + \frac{1}{6} \varepsilon^\lambda{}_{\rho\mu\nu} S^\rho + t^\lambda{}_{\mu\nu},$$

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New question to address

Is it possible to have both torsion and nonmetricity propagating in a stable theory?

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- The first contribution comes from mixing terms which depend linearly on the curvature tensor and quadratically on the torsion tensor.

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- There are 46 possible terms with couplings constants from $\{h_i\}^{i=1,..46}$ that can be split as:

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- For convenience, we have introduced the Lagrangian coefficients of this branch to match the notation used in the absence of nonmetricity.

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- There are 61 possible terms with couplings constants from $\{h_i\}^{i=47,\dots,107}$ that can be split as:

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- The last contributions comes from mixing terms which depend linearly on the curvature tensor and linearly on both torsion and nonmetricity tensors.
- That case is extremely non-trivial since there are 102 different terms with couplings constants from $\{h_i\}^{i=108,\dots,209}$:

$$\begin{aligned} \mathcal{L}_{\text{curv-tor-nonm}}^{(3)} = & \mathcal{L}_{\tilde{R}T W}^{(3)} + \mathcal{L}_{\tilde{R}T\Lambda}^{(3)} + \mathcal{L}_{\tilde{R}T\Omega}^{(3)} + \mathcal{L}_{\tilde{R}T q}^{(3)} + \mathcal{L}_{\tilde{R}S W}^{(3)} + \mathcal{L}_{\tilde{R}S\Lambda}^{(3)} + \mathcal{L}_{\tilde{R}S\Omega}^{(3)} + \mathcal{L}_{\tilde{R}S q}^{(3)} + \mathcal{L}_{\tilde{R}t W}^{(3)} \\ & + \mathcal{L}_{\tilde{R}t\Lambda}^{(3)} + \mathcal{L}_{\tilde{R}t\Omega}^{(3)} + \mathcal{L}_{\tilde{R}t q}^{(3)}, \end{aligned}$$

where for example the first term reads

$$\begin{aligned} \mathcal{L}_{\tilde{R}T W}^{(3)} = & h_{108} \tilde{R}_{\alpha\rho} T^\rho W^\alpha + h_{109} \tilde{R}_{\alpha\rho} T^\alpha W^\rho + h_{110} \hat{R}_{\alpha\rho} T^\rho W^\alpha + h_{111} \hat{R}_{\alpha\rho} T^\alpha W^\rho + h_{112} \tilde{R}^\omega{}_\omega{}^\alpha{}_\rho T_\alpha W^\rho \\ & + h_{113} \tilde{R} T_\alpha W^\alpha. \end{aligned}$$

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- In order to understand the stability of the theory, it is useful to define the following kinetic terms as

$$\mathcal{L}_{FF} = -\frac{1}{4}\kappa_{XY}F_{\mu\nu}^{(X)}F^{(Y)\mu\nu}, \quad F^{(X)} = (F^{(S)}, F^{(T)}, F^{(W)}, F^{(\Lambda)}),$$

where the kinetic matrix becomes

$$\kappa_{XY} = -2 \begin{pmatrix} 2l_6 & 0 & 0 & 0 \\ 0 & 2l_1 & l_{30} & l_{43} \\ 0 & l_{30} & 2l_{11} & l_{70} \\ 0 & l_{43} & l_{70} & 2l_{16} \end{pmatrix}.$$

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- We need to eliminate all possible pathological terms and then study the kinetic matrix.

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- After noticing this, we reduce the kinetic matrix for the whole remaining system as a 3x3 matrix one given by

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- Again, this matrix must be non-negative and since $-4l_1 \geq 0$, we can split the study into three different 2x2 matrices associated to the pairs $\{T_\mu, W_\mu\}$, $\{T_\mu, \Lambda_\mu\}$ and $\{W_\mu, \Lambda_\mu\}$.

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- One needs to diagonalise such kinetic matrices and then assume that all eigenvalues are non-negative. This is indeed possible to do and then, we find that both torsion and nonmetricity can be propagating in a healthy way.

Reissner-Nordström-like black holes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0.$$

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- Static and spherically symmetric space-times:

$$\#10 \rightarrow \#2 \left\{ ds^2 = \Psi_1(r) dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) ; \right.$$

$$\#24 \rightarrow \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \rightarrow \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

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- It is important to mention that in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero \implies We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

Conclusions

- The well-known instabilities in the vector and axial sectors of quadratic Poincaré gauge theory (torsion cannot be propagating) and also Metric-Affine gravity can be eliminated by introducing cubic order invariants in the gravitational action.

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- In analogy to quadratic MAG theories, a further restriction on the Lagrangian coefficients provides Reissner-Nordström-like black hole solutions with dynamical torsion and nonmetricity.
- For the first time we found a solution with all the intrinsic charges without known instabilities and with all higher-spin masses of torsion and nonmetricity.

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- It is possible to find a renormalizable theory with these cubic interactions?