Metric-Affine Theories of Gravity: From theory to new possible effects

Sebastián Bahamonde

Postdoctoral Researcher at Kavli IPMU, University of Tokyo, Japan

Visiting Researcher at Institute for Basic Science, Daejeon, South Korea.
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Mainly jointly with Jorge Gigante Valcarcel

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Outline

1. Metric-Affine gravity and Gauge approach with Cubic interactions
2. Black holes with torsion and nonmetricity
   - Spherically symmetric black holes
   - Axially symmetric black holes
3. Cosmology with torsion and nonmetricity
4. Main results

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  - New geometric degrees of freedom arise:

$$T^{\lambda}{}_{\mu\nu} = 2\tilde{\Gamma}^{\lambda}{}_{[\mu\nu]} \quad 24 \text{ dof} \quad \text{(Measures the nonclosure of infinitesimal parallelograms)}$$

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  - The general curvature is defined as in GR but changing $\Gamma^{\lambda}{}_{\mu\nu}$ to $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = \partial_{\mu}\tilde{\Gamma}^{\lambda}{}_{\rho\nu} - \partial_{\nu}\tilde{\Gamma}^{\lambda}{}_{\rho\mu} + \tilde{\Gamma}^{\lambda}{}_{\sigma\mu}\tilde{\Gamma}^{\sigma}{}_{\rho\nu} - \tilde{\Gamma}^{\lambda}{}_{\sigma\nu}\tilde{\Gamma}^{\sigma}{}_{\rho\mu}$$

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

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- Curvatures and field strengths:

$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac} g_{bd} e^{c\lambda} e^{d\rho} Q_{\mu\lambda\rho}\,,$$
$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\,e^b{}_\nu - \omega^a{}_{b\nu}\,e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,$$
$$F^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu}\,\omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\,\omega^c{}_{b\mu} = g_{bc}\,e^a{}_\lambda e^{c\rho} \tilde{R}^\lambda{}_{\rho\mu\nu}\,.$$

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- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity.

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S=\int d^4x\sqrt{-g}\Big[\mathcal{L}_m-\frac{1}{16\pi}\mathcal{L}_g\big(\tilde{\mathcal{R}},\mathcal{T},\mathcal{Q}\big)\Big]\,.$$

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- Correspondence between geometry and matter:

$$\begin{aligned}
\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu\,,\\
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- Hypermomentum can be split into three parts:

$$\Delta_{\mu\nu\lambda}={}^{(\mathrm{s})}\Delta_{[\mu\nu]\lambda}+\frac{1}{4}g_{\mu\nu}{}^{(\mathrm{d})}\Delta_\lambda+{}^{(\mathrm{sh})}\,\check{\Delta}_{(\mu\nu)\lambda}$$

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Quadratic Poincaré gauge theory - ghost issue

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+\, d_1\tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\,d^4x\,.$$

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$$+ d_1 \tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2 T_{\mu} T^{\mu} + m_S^2 S_{\mu} S^{\mu} + m_t^2 t_{\lambda\mu\nu} t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\, d^4x\,.$$

- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_{μ} and S_{μ} propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^{\mu} T^{\nu} + h_2 \tilde{R} T_{\mu} T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^{\mu} S^{\nu} + h_4 \tilde{R} S_{\mu} S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_{\sigma} S^{\sigma} + h_{14} \varepsilon_{\nu}{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^{\mu} S^{\nu} + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_{\mu} S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu},\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t^{\alpha\tau}{}_{\gamma} + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau}\\
&+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^{\rho}{}_{\mu\tau\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\gamma\rho\tau}\\
&+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^{\mu}{}_{\gamma} S^{\alpha} t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_{\sigma} t^{\sigma\mu\nu}\,.
\end{aligned}$$

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- We showed that by including these Poincaré gauge invariants, ghost issue is solved!

Cubic Metric-Affine gauge theory

- Let us focus on the axial and vector sectors. Let us recall the decomposition:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,,$$

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Cubic Metric-Affine gravity Lagrangian

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- We showed that by introducing these terms that are part of the Gauge approach, one can eliminate all known vector/axial ghosts.

Outline

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

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$$\#40 \to \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

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$$g_{tt} = -\frac{1}{g_{rr}} = \Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(H_1\kappa_{\rm s}^2 + H_2\kappa_{\rm d}^2 + H_3\kappa_{\rm sh}^2\right).$$

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- It is important to mention that the in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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5. The intrinsic shear charge $\kappa_{\rm d}$: deformations that change the shape of an object without changing its volume.

Electrodynamics coupled with torsion

- Now, consider another theory with couplings between the electromagnetic field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $\tilde{R}^{\lambda}{}_{\rho\mu\nu}$: (S. Bahamonde, J. Maggiolo and C. Pfeifer, arXiv:2507.02362)

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- New coupling between intrinsic spin charge κ_s and electric charge q.
- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

3D Axially symmetric black holes

- In 3D, Torsion has 9 dof: In the electromagnetism with torsion theory we found a slowly BTZ black hole solution:

$$ds^2 = \Big(f_A(r) + J f_B(r)\Big)dt^2 - \Big(\frac{1}{f_A(r)} + J\,\frac{f_B(r)}{f_A^2}\Big)dr^2 - r^2 d\phi^2$$
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- We notice a new coupling between the intrinsic spin $\kappa_{\rm s}$, the angular momentum of the rotation J and the magnetic charge q_m, i.e. $\propto J q_m \kappa_{\rm s}$.

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

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- In this case, this would be an interaction between the angular momentum a and the intrinsic spin κ_s in 4D.

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- The form of torsion contains 24 dof being non-zero.
- Is there any interesting new effect that can emerge from this analogy?

Outline

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0\,.$$

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -\,N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2 \gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\,\bar{n}_{[\mu}\bar{P}_{\nu]}{}^\lambda + 2T_2(t)\,\bar{\varepsilon}^{\,\lambda}{}_{\mu\nu\rho}\bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t)\bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t)\bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t)\bar{P}_{\lambda(\mu}\bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

Cosmology in MAG - matter

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

$$T_{\mu\nu} = (\rho(t) + p(t))\, n_\mu n_\nu + p(t) g_{\mu\nu} = \rho(t) n_\mu n_\nu + p(t) p_{\mu\nu}\,,$$

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3}\Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4}\Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho \,,$$

which contains 5 different sources dof related to the intrinsic spin, dilations, and shears.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_\mu T^\mu{}_\alpha - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_\mu\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\alpha{}^{\mu\nu}) + 2T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) = 0\,.$$

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- In our Poincaré Gauge Gravity theory, we have the modified FLRW equations of the form: S. Bahamonde, R. Briffa, K. Dialektopoulos, D. Iosifidis and J. Levi Said, arXiv:2506.17017.

$$3H^2 + f(T_1(t), T_2(t)) = \kappa^2\rho\,, \quad 3H^2 + 2\dot{H} + g(T_1(t), T_2(t)) = -\kappa^2 p\,.$$

and the connection equations:

$$2\kappa^2\,{}^{(\mathrm{s})}\Delta_3 = 3T_1\Big[6h_1(H-T_1)(H-2T_1) - 6(h_1 - 16h_{13})T_2^2 + m_T^2\Big]\,,$$

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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T.

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- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

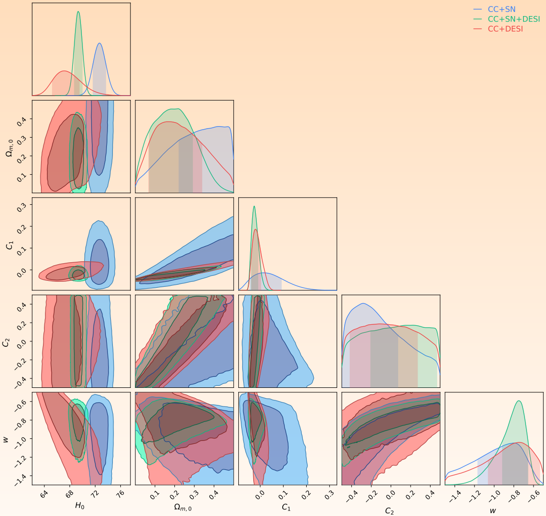

Figure: Confidence contours and posterior distributions for the Independent conservation of fluid and hypermomentum model parameters which include C_1, C_2 and w. H_0 and $\Omega_{m,0}$ are also constrained. The blue contours represent the CC+SN dataset, the red contours illustrate CC+DESI, while the green contours correspond to the combination CC+SN+DESI.

SVT decomposition around FLRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(\mathrm{T})}, h_i^{(\mathrm{T})}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(\mathrm{TT})}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(\mathrm{T})}, B_i^{(\mathrm{T})}, A_i^{(\mathrm{T})}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(\mathrm{T})}, \mathcal{B}_i^{(\mathrm{T})}, \mathcal{A}_i^{(\mathrm{T})}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(\mathrm{TT})}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(\mathrm{TT})}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, “Cosmological Perturbation Theory in Metric-Affine Gravity,” Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FLRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\mathrm{T})}{}_i, Y^{(\mathrm{T})}{}_i, Z^{(\mathrm{T})}{}_i, \kappa^{(\mathrm{T})}{}_i, Q^{(\mathrm{T})}{}_i, W^{(\mathrm{T})}{}_i, C^{(\mathrm{T})}{}_i\}$	2 dof each	14
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- Since we have spin-3,spin-2,spin-1,spin-0 being dynamical, different effects might emerge! (like Iman's talk)

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- In cosmology:
  - Torsion and hypermomentum lead to **faster expanding cosmologies** with similar matter content as ΛCDM. **Geometry acts as effective curvature, radiation or varying** G.

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  - Modified Einstein–Maxwell theory: $k_3 F^{\mu\nu}\tilde{R}_{\mu\nu}$ with interactions $q_e\kappa_s$ (4D) and $q_m\kappa_s a$ (3D).
  - Cubic MAG: dynamical T, Q, massive spin-2 and spin-3 with intrinsic charges: **spin, dilation, shear**.
  - Cubic Poincaré: slowly rotating Kerr-like BH with **gravitational spin-orbit interaction**: $\mathcal{L} \propto \frac{1}{16\pi G r^6}\, a d_1 \kappa_s m(\alpha\cos\vartheta + \beta\sin\vartheta)$.
- In cosmology:
  - Torsion and hypermomentum lead to **faster expanding cosmologies** with similar matter content as ΛCDM. **Geometry acts as effective curvature, radiation or varying** G.
  - Developed **cosmological perturbation theory** including helicity-3 sector of spin-3 nonmetricity.