Theories with torsion and nonmetricity: basic concepts and what's next

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Outline

Overview of the Talk

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- By doing that, the metric is no longer the only independent quantity, but now the general affine connection $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$ is also independent.
- In the most general scenario, geometry contains curvature, torsion and nonmetricity. Those geometries are labelled as “Metric-Affine geometries.

Definitions and conventions

- The metric tensor $g_{\mu\nu}$ constitutes a natural isomorphism between the tangent and cotangent spaces defined at any point of the differentiable manifold

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- Thus it allows the measurement of infinitesimal distances

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \,,$$

as well as of vector lengths and angles among them

$$||\mathbf{U}|| = \sqrt{|g_{\mu\nu} U^\mu U^\nu|}, \quad \cos\alpha = \frac{g_{\mu\nu} m^\mu n^\nu}{\sqrt{|g_{\alpha\beta} m^\alpha m^\beta|}\sqrt{|g_{\gamma\sigma} n^\gamma n^\sigma|}} \,.$$

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- The affine/linear connection $\tilde{\Gamma}^{\rho}{}_{\lambda\mu}$ defines the covariant derivative operator

$$\tilde{\nabla}_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \tilde{\Gamma}^{\nu}{}_{\lambda\mu}V^{\lambda}\,,$$
$$\tilde{\nabla}_{\mu}V^{\nu} \xrightarrow{GCT} \tilde{\nabla}'_{\mu}V'^{\nu} = \Lambda_{\mu}{}^{\lambda}\Lambda^{\nu}{}_{\rho}\tilde{\nabla}_{\lambda}V^{\rho}\,,$$

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- Commutation rule:

$$[\tilde{\nabla}_{\mu}, \tilde{\nabla}_{\nu}]\,v^{\lambda} = \tilde{R}^{\lambda}{}_{\rho\mu\nu}\,v^{\rho} + T^{\rho}{}_{\mu\nu}\,\tilde{\nabla}_{\rho}v^{\lambda}\,.$$

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- In these new variables, one can easily define spinors (as in GR) but one of the main advantage is that the curvature form DO NOT depend on the tetrad field (or metric):

$$F^{a}{}_{b\mu\nu} = \partial_{\mu}\omega^{a}{}_{b\nu} - \partial_{\nu}\omega^{a}{}_{b\mu} + \omega^{a}{}_{c\mu}\,\omega^{c}{}_{b\,\nu} - \omega^{a}{}_{c\nu}\,\omega^{c}{}_{b\mu}\,.$$

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- Recall that one can ”go back” to the standard curvature by contracting this quantity with tetrads:

$$\tilde{R}^\lambda{}_{\rho\mu\nu} = e_a{}^\lambda e^b{}_\rho \, F^a{}_{b\mu\nu}$$

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- In particular, it measures the change of vector components on parallel transport along an infinitesimal closed curve:

$$\delta V_{\mu} = \tilde{R}^{\lambda}{}_{\mu\rho\nu} V_{\lambda} ds^{\rho\nu} \,,$$

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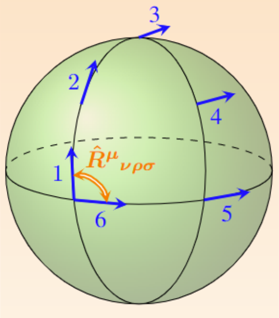

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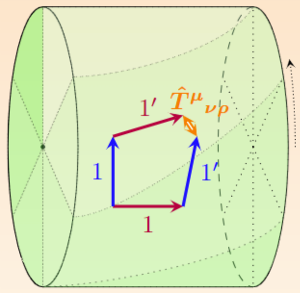

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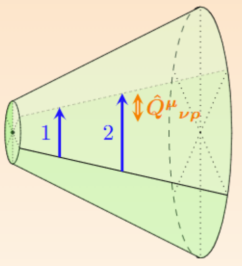

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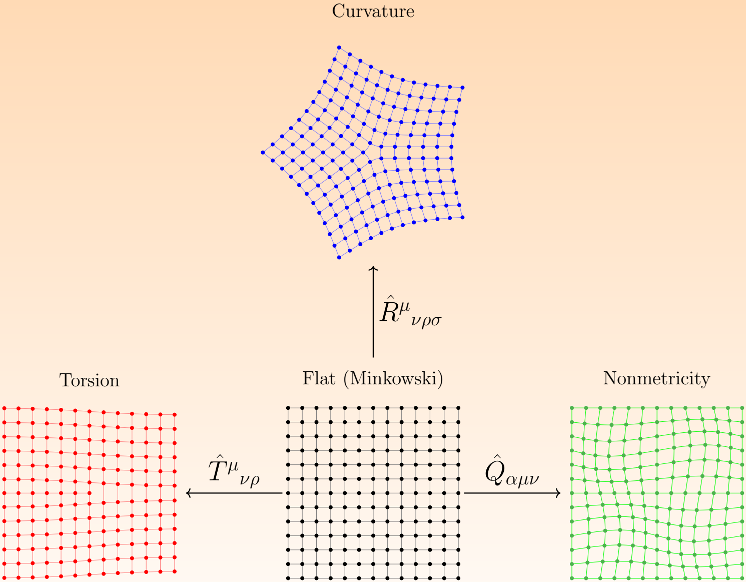
- **Nonmetricity:** can be understood as crystalline structure with point defects (vacancies/intersticials)



ure: Crystalline structure and its analogy with curvature, torsion and
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$$\Delta_{\mu\nu\lambda} = {}^{(\mathrm{s})}\Delta_{[\mu\nu]\lambda} + \frac{1}{4} g_{\mu\nu}{}^{(\mathrm{d})}\Delta_{\lambda} + {}^{(\mathrm{sh})}\not{\Delta}_{(\mu\nu)\lambda}\,.$$

Here, the first part is related to intrinsic spin (which is the source of torsion), the second part is related to intrinsic dilations (which is the source of the trace part of nonmetricity) with ${}^{(\mathrm{d})}\Delta_{\mu} = \Delta^{\nu}{}_{\nu\mu}$, and the last part is related to intrinsic shears (which provide a source for the traceless part of nonmetricity).

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- Remember that they are INTRINSIC properties of the geometry. Intrinsic spin $\neq$ macroscropic spin (for example Kerr angular momentum)

Example: Dirac equation in the presence of torsion

- Covariant derivative of a Dirac spinor and its adjoint

$$\tilde{\nabla}_\mu \Psi = \partial_\mu \Psi - \frac{1}{8}\,\omega^{ab}{}_{\mu}\,[\gamma_a, \gamma_b]\,\Psi\,,$$

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- The Dirac Lagrangian provides the dynamics of $1/2$ spin fields under minimal coupling

$$\tilde{\mathcal{L}}_{\rm Dirac} = \frac{i}{2}\left(\bar{\Psi}\,\gamma^\mu \tilde{\nabla}_\mu \Psi - \tilde{\nabla}_\mu \bar{\Psi}\,\gamma^\mu \Psi - 2im\bar{\Psi}\Psi\right) \,.$$

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- In the framework of post-Riemannian geometry

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- Accordingly, only the axial mode of torsion interacts with $1/2$ fields under minimal coupling

$$\tilde{\mathcal{L}}_{\text{Dirac}} = \mathcal{L}_{\text{Dirac}} - \frac{1}{8}\,\varepsilon^{\lambda\rho\mu\nu} T_{\lambda\rho\mu} \bar{\Psi}\gamma^5 \gamma_\nu \Psi\,.$$

Symmetries and identities of the curvature tensor

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\tilde{\nabla}_{[\sigma|}\tilde{R}^{\lambda}{}_{\rho|\mu\nu]} &= T^{\omega}{}_{[\sigma\mu|}\tilde{R}^{\lambda}{}_{\rho\omega|\nu]}\,,\\
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- Three independent rank-2 tensors defined from the first contractions of the curvature tensor:

$$\tilde{R}_{\mu\nu} = \tilde{R}^{\lambda}{}_{\mu\lambda\nu}\,, \quad \hat{R}_{\mu\nu} = \tilde{R}_{\mu}{}^{\lambda}{}_{\nu\lambda}\,, \quad \tilde{R}^{\lambda}{}_{\lambda\mu\nu} = \nabla_{[\nu} Q_{\mu]\lambda}{}^{\lambda}\,.$$

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- Unique scalar and pseudoscalar curvatures:

$$\tilde{R} = g^{\mu\nu} \tilde{R}_{\mu\nu}\,, \quad *\tilde{R} = \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu}\,.$$

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}\,,$$

with

$$N^{\lambda}{}_{\mu\nu} = K^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}\,,$$

where the contortion $K^{\lambda}{}_{\mu\nu}$ and disformation tensors $L^{\lambda}{}_{\mu\nu}$ are

$$K^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left(T^{\lambda}{}_{\mu\nu} - T_{\mu}{}^{\lambda}{}_{\nu} - T_{\nu}{}^{\lambda}{}_{\mu}\right)\,,$$

$$L^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left(Q^{\lambda}{}_{\mu\nu} - Q_{\mu}{}^{\lambda}{}_{\nu} - Q_{\nu}{}^{\lambda}{}_{\mu}\right)\,.$$

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}\,,$$

with

$$N^{\lambda}{}_{\mu\nu} = K^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}\,,$$

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- General decomposition of the the curvature tensor:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|}N^{\lambda}{}_{\rho|\nu]} + 2N^{\lambda}{}_{\sigma[\mu|}N^{\sigma}{}_{\rho|\nu]}\,.$$

Overview of the Talk

Dynamics and action

- One can write down an action of the form

$$S = \int d^4x\sqrt{-g}\Big[\mathcal{L}_m - \frac{1}{2\kappa^2}\mathcal{L}_g(\tilde{\mathcal{R}}, \mathcal{T})\Big]\,.$$

with $\kappa^2 = 8\pi G$.

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with $\kappa^2 = 8\pi G$.

- Correspondence between geometry and matter:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 2\kappa^2\theta_a{}^\nu\,,\\ \frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta \omega^a{}_{b\nu}} &= 2\kappa^2\Delta_a{}^{b\nu}\,,\end{aligned}$$

where we have two matter sources:

$$\begin{aligned}\theta_\mu{}^\nu &= \frac{e^a{}_\mu}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\rm m}\right)}{\delta e^a{}_\nu}\,,\\ \Delta^{\lambda\mu\nu} &= \frac{e^{a\lambda}e_b{}^\mu}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_{\rm m}\right)}{\delta\omega^a{}_{b\nu}}\,.\end{aligned}$$

Conservation equation

- $\theta_a{}^{\nu}$ is the canonical energy-momentum tensor that can be related to the energy-momentum tensor as

$$\Theta^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} + \frac{1}{2\sqrt{-g}}\Big[\tilde{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu}) - 2\sqrt{-g}\,T_{\nu}\Delta_{\lambda}{}^{\mu\nu}\Big]\,.$$

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- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation:

$$\nabla_{\mu}T^{\mu}{}_{\alpha} - \frac{1}{2}\Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha} + \frac{1}{2}\tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}\Delta_{\alpha}{}^{\mu\nu} + T_{\mu\alpha}{}^{\lambda}\tilde{\nabla}_{\nu}\Delta_{\lambda}{}^{\mu\nu} = 0\,.$$

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- Obviously, if there is no hypermomentum, we find the standard energy-momentum conservation $\nabla_{\mu}T^{\mu}{}_{\alpha} = 0$.

Poincaré gauge theory

- $ISO(1,3) = T^4 \rtimes SO(1,3)$ gauge connection[1]:

$$A_\mu = e^a{}_\mu P_a + \omega^a{}_{b\mu} J_a{}^b \,,$$
$$g_{\mu\nu} = e^a{}_\mu e^b{}_\nu \eta_{ab} \,,$$

where

$$\omega^a{}_{b\mu} = e^a{}_\lambda \, e_b{}^\rho \, \tilde{\Gamma}^\lambda{}_{\rho\mu} + e^a{}_\lambda \, \partial_\mu \, e_b{}^\lambda \,.$$

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$$\omega^a{}_{b\mu} = e^a{}_\lambda \, e_b{}^\rho \, \tilde{\Gamma}^\lambda{}_{\rho\mu} + e^a{}_\lambda \, \partial_\mu \, e_b{}^\lambda \,.$$

- Generators of the Poincaré group $ISO(1,3)$:

$$[P_a, P_b] = 0 \,,$$
$$[P_a, J_{bc}] = i \, \eta_{a[b} \, P_{c]} \,,$$
$$[J_{ab}, J_{cd}] = \frac{i}{2} \left(\eta_{ad} \, J_{bc} + \eta_{cb} \, J_{ad} - \eta_{db} \, J_{ac} - \eta_{ac} \, J_{bd} \right) \,.$$

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Quadratic Poincaré gauge theory

- $ISO(1,3)$ translational and rotational field strength tensors:

$$F_{\mu\nu} = F^a{}_{\mu\nu} P_a + F^a{}_{b\mu\nu} J_a{}^b \,,$$

where

$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\, e^b{}_\nu - \omega^a{}_{b\nu}\, e^b{}_\mu \,,$$
$$F^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu}\, \omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\, \omega^c{}_{b\mu} \,.$$

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$$F^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu}\, \omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\, \omega^c{}_{b\mu} \,.$$

- Recall the correspondence with the torsion and curvature tensors:

$$F^a{}_{\mu\nu} = e^a{}_\lambda T^\lambda{}_{\nu\mu} \,,$$
$$F^a{}_{b\mu\nu} = e^a{}_\lambda e_b{}^\rho \tilde{R}^\lambda{}_{\rho\mu\nu} \,.$$

Overview of the Talk

Fallacies about Torsion 1 (book Gauge Theories of Gravitation", M. Blagojevic and F.W. Hehl)

- Carroll[2] argues on p. 190 of his book said:

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Fallacy 1: Torsion not distuinsguied from other tensor fields

"We could drop the demand that the connection be torsion-free, in which case the torsion tensor could lead to additional propagating degrees of freedom. The basic reason why such theories do not receive much attention is simply because the torsion is itself a tensor; there is nothing to distinguish it from other, non gravitational tensor fields. Thus, we do not really lose any generality by considering theories of torsion-free connections (which lead to GR) plus any number of tensor fields, which we can name what we like. Similar considerations apply when we consider dropping the requirement of metric compatibility—any connection can be written as a metric-compatible connection plus a tensorial correction, so any such theory is equivalent to GR plus extra tensor fields, which wouldn't really deserve to be called an "alternative to general relativity"

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- The closure failure of a parallelogram can only be created by a distinctive geometrical quantity, namely the torsion tensor, and not by any other tensor.
- Moreover, torsion affects the Bianchi identities

Fallacies about Torsion 2 (book Gauge Theories of Gravitation", M. Blagojevic and F.W. Hehl)

Is there any physical principle . . .?

Weinberg claimed "...Sorry, I still don't get it (on the possibility of torsion). Is there any physical principle, such as a principle of invariance, that would require the Christoffel symbol to be accompanied by some specific additional tensor? Or that would forbid it? And if there is such a principle, does it have any other testable consequences?"

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- The physical principle Weinberg is looking for is translational gauge invariance.
- The testable consequences are related to the new spin-spin contact interaction and to the precession of elementary particle spins in torsion fields.

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Gravity Probe B and torsion

Torsion can be measured (and constraint a lot) by means of the Gravity Probe B experiment

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- Then, it turns out that measuring torsion requires elementary spin, there is no other way. Orbital angular momentum is not a substitute for spin.
- Would you use an electrically and magnetically neutral test particle for tracing possible electromagnetic fields?
- By the same token: Would you use a macroscopically rotating quartz ball of Gravity Probe B for tracing a possible torsion field?

Overview of the Talk

Quadratic Poincaré gauge theory - ghost issue

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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  - Axial vector part $S_{\mu} = \varepsilon_{\mu\nu\rho\sigma}T^{\nu\sigma\rho}$,
  - Tensor part $t^{\lambda}{}_{\mu\nu} = T^{\lambda}{}_{\mu\nu} - \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) - \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho}$.

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+\, d_1\tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\,d^4x\,.$$

Quadratic Poincaré gauge theory - ghost issue

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- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_{μ} and S_{μ} propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms:[3]

$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^\mu T^\nu + h_2 \tilde{R} T_\mu T^\mu\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^\mu S^\nu + h_4 \tilde{R} S_\mu S^\mu\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_\sigma t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_\sigma t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_\sigma t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^\lambda t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^\lambda t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_\sigma S^\sigma + h_{14} \varepsilon_\nu{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^\mu S^\nu + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_\mu S_\nu\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^\nu t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^\rho t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_\mu t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_\mu t^{\lambda\rho\mu},\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau{}^{\rho\mu\nu} S^\gamma t^{\alpha\tau}{}_\gamma + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau{}^{\rho\mu\nu} S^\gamma t_\gamma{}^{\alpha\tau}\\
&+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^\rho{}_{\mu\tau\nu} S^\gamma t_\gamma{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\gamma\rho\tau}\\
&+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^\mu{}_\gamma S^\alpha t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_\sigma t^{\sigma\mu\nu}\,.
\end{aligned}$$

[3] S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10

Vector/Axial in cubic Poincaré gauge theory (with $t^{\lambda}{}_{\mu\nu} = 0$)

- First stability constraints (6 equalities):

$$c_2 = 2c_1 \,, \quad h_2 = -\frac{h_1}{2} \,, \quad h_3 = -\frac{1}{6}\left(c_1 + 6h_{13}\right) \,,$$
$$h_4 = \frac{h_{13}}{2} \,, \quad h_{14} = -2h_{13} \,, \quad h_{15} = 4h_{13} \,.$$

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- Second stability constraints (inequalities):
  - Kinetic terms:

$$-\frac{d_1}{6} \leq c_1 \leq -\frac{d_1}{3}\,, \quad d_1 \leq 0\,.$$

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  - Kinetic terms:

$$-\frac{d_1}{6} \leq c_1 \leq -\frac{d_1}{3}\,, \quad d_1 \leq 0\,.$$

  - Quartic order of the potential:

$$h_{13} \leq \frac{h_1}{16} \leq \left(19 + 6\sqrt{10}\right)h_{13}\,, \quad h_{13} < 0\,.$$

Vector/Axial in cubic Poincaré gauge theory (with $t^{\lambda}{}_{\mu\nu} = 0$)

- First stability constraints (6 equalities):

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$$m_T^2 \geq 0\,, \quad m_S^2 \geq 0\,.$$

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- Is it possible to find a renormalizable theory with these cubic interactions?

Overview of the Talk

Spherical symmetry

- Explicit symmetries on the metric, torsion and nonmetricity tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

$$\#24 \to \#8 \left\{ \begin{array}{ccc} T^{t}{}_{tr} & T^{r}{}_{tr} & T^{\vartheta}{}_{t\vartheta} \\ T^{\vartheta}{}_{r\vartheta} & T^{\vartheta}{}_{t\varphi} & T^{\vartheta}{}_{r\varphi} \\ T^{t}{}_{\vartheta\varphi} & T^{r}{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \to \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

Reissner-Nordström-like black holes

- Metric and torsion tensors of the solution:

$$T^t{}_{tr} = \frac{\Psi'(r)}{2\Psi(r)} + \frac{wr}{\Psi(r)}\,, \quad T^r{}_{tr} = T^t{}_{tr}\Psi(r)\,,$$

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$$T^\vartheta{}_{t\varphi} = -T^\vartheta{}_{r\varphi}\Psi(r)\,, \quad T^\vartheta{}_{r\varphi} = \frac{N_1\kappa_{\rm s}}{r\Psi(r)}\sin\vartheta\,,$$

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$$\Psi_1(r) = \Psi_2(r) = \Psi(r)\,,$$

where $\Psi(r) = 1 - \frac{2m}{r} + (2N_1 - N_2)(N_1 + N_2)\left[\frac{2N_1+N_2}{4N_1+N_2}d_1 + 2h_{25}\right]\frac{\kappa_{\rm s}^2}{3r^2}$ and $w = -\frac{(4N_1+N_2)(2N_1+5N_2)}{6(2N_1-N_2)[d_1(2N_1+N_2)+(h_{25}-h_6)(4N_1+N_2)]}m_t^2$.

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- Reduced vector and axial sectors:

$$\mathcal{L}_g = \frac{1}{16\pi}\left[-R + \frac{2}{9}(3c_1 + d_1)F^{(T)}_{\mu\nu}F^{(T)\mu\nu} - \frac{1}{72}(6c_1 + d_1)F^{(S)}_{\mu\nu}F^{(S)\mu\nu}\right].$$

Electrodynamics coupled with torsion

- the most general extension theory that is parity-preserving and satisfies that the extended Lagrangian is linear in curvature $\tilde{R}^{\lambda}{}_{\rho\mu\nu}$ and in the field strength $F_{\mu\nu}$, and quadratic in torsion $T^{\lambda}{}_{\mu\nu}$, is:

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- We found the following black hole solution:

$$t_1(r) = \frac{\Psi'(r)}{2\Psi(r)} + \frac{c_1}{\Psi(r)} + \frac{\kappa_{\mathrm{s}}}{r\Psi(r)}\,, \quad t_2(r) = \Psi(r)\Big(t_1(r) - t_4(r) - \frac{2}{r}\Big)\,, \tag{1}$$

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- New coupling between spin charge κ_s and electric charge q.
- Different charges would give rise to different phenomenology.

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3. The solution is in vacuum and a charge κ_s appears (spin charge). Analogue to the case of Schwarzschild where the mass M appears.
4. We expect that the spin charge might be important in certain astrophysical scenarios such as: highly magnetized neutron stars; supermassive black holes with endowed spin.

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- We recently found a solution in the non-dynamical part and with that part, we are investigating the dynamical part.
- In spherical symmetry we obtained RN, but in axial symmetry we expect to have an extension of Kerr-Newman with new interactions between κ_s (intrinsic spin) and macroscropic spin a.

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- Another possibility is that in gravity, the gravitational spin-orbit interaction would not be of the form $a\kappa_s$ but $a^2\kappa_s$ or $a\kappa_s^2$.

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- Study neutron stars and study the effect of the intrinsic spin in such scenarios.

Overview of the Talk

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- The majority of these studies predominantly focus on torsion.
- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\,\bar{n}_{[\mu}\bar{P}_{\nu]}{}^\lambda + 2T_2(t)\,\bar{\varepsilon}^{\,\lambda}{}_{\mu\nu\rho}\bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t)\bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t)\bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t)\bar{P}_{\lambda(\mu}\bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

Cosmology in MAG - matter

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$$T_{\mu\nu} = (\rho(t) + p(t))\, n_\mu n_\nu + p(t) g_{\mu\nu} = \rho(t) n_\mu n_\nu + p(t) p_{\mu\nu}\,,$$

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- For Cubic Poincare we found the following energy-conservation:

$$\dot{\rho} + 3H(\rho + p) = 3\, {}^{(\mathrm{s})}\Delta_5 \Big(H T_2 + \dot{T}_2 \Big) - 3\, {}^{(\mathrm{s})}\Delta_3 \Big(\dot{H} - H T_1 + H^2 - \dot{T}_1 \Big)\ .$$

SVT decomposition around FRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(\mathrm{T})}, h_i^{(\mathrm{T})}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(\mathrm{TT})}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(\mathrm{T})}, B_i^{(\mathrm{T})}, A_i^{(\mathrm{T})}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(\mathrm{T})}, \mathcal{B}_i^{(\mathrm{T})}, \mathcal{A}_i^{(\mathrm{T})}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(\mathrm{TT})}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(\mathrm{TT})}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\mathrm{T})}{}_i, Y^{(\mathrm{T})}{}_i, Z^{(\mathrm{T})}{}_i, \kappa^{(\mathrm{T})}{}_i, Q^{(\mathrm{T})}{}_i, W^{(\mathrm{T})}{}_i, C^{(\mathrm{T})}{}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(\mathrm{T})}{}_i, \mathcal{Q}^{(\mathrm{T})}{}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(\mathrm{TT})}{}_{ij}, Q^{(\mathrm{TT})}{}_{ij}, C^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	6
1 rank-2 pseudotensor	$\{\mathcal{Q}^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	2
1 rank-3 tensor	$\{C^{(\mathrm{TT})}{}_{ijk}\}$	2 dof each	2

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- We studied this spin-3 in quadratic MAG finding

$$\ddot{C}^{(A)} + 3H\dot{C}^{(A)} + [\omega_{C,k}^{(A)}]^2 C^{(A)} = J^{(A)} .$$

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- The massive spin-3 part of nonmetricity might be a good candidate for dark matter. This has not been addressed in detail.

Thank You for Listening!

Questions? Comments?

Join us to work in all of these questions!