# Foundations of Metric-Affine Theories of Gravity and black holes

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# Outline

1. Introduction to metric-affine geometry
2. Teleparallel theories
   - Trinity of Gravity
   - Modified Theories with torsion and applications (Metric TG)
   - Theories with Nonmetricity and applications (Symmetric TG)
3. Metric-Affine theories (where curvature is non-vanishing)
4. Black holes with torsion and nonmetricity
   - Spherically symmetric black holes
   - Axially symmetric black holes

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# Definitions and conventions

- The affine/linear connection $\tilde{\Gamma}^{\rho}{}_{\lambda\mu}$ defines the covariant derivative operator

$$\tilde{\nabla}_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \tilde{\Gamma}^{\nu}{}_{\lambda\mu}V^{\lambda}\,,$$

$$\tilde{\nabla}_{\mu}V^{\nu} \xrightarrow{GCT} \tilde{\nabla}'_{\mu}V'^{\nu} = \Lambda_{\mu}{}^{\lambda}\Lambda^{\nu}{}_{\rho}\tilde{\nabla}_{\lambda}V^{\rho}\,,$$

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- In general, the metric tensor and the affine connection are independent quantities.
- Commutation rule:

$$[\tilde{\nabla}_{\mu}, \tilde{\nabla}_{\nu}]\, v^{\lambda} = \tilde{R}^{\lambda}{}_{\rho\mu\nu}\, v^{\rho} + T^{\rho}{}_{\mu\nu}\, \tilde{\nabla}_{\rho}v^{\lambda}\,.$$

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- In particular, it measures the change of vector components on parallel transport along an infinitesimal closed curve:

$$\delta V_{\mu} = \tilde{R}^{\lambda}{}_{\mu\rho\nu} V_{\lambda} ds^{\rho\nu}\,,$$

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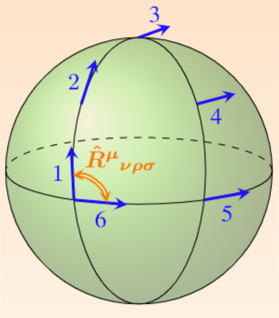

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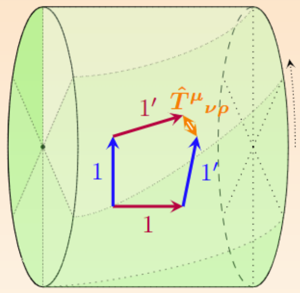


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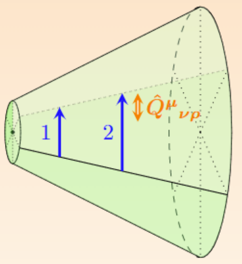


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# Post-Riemannian decomposition

- According to the Fundamental Theorem of Riemannian geometry, in the absence of torsion and nonmetricity:

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho} \left( \partial_{\mu} g_{\nu\rho} + \partial_{\nu} g_{\mu\rho} - \partial_{\rho} g_{\mu\nu} \right) ,$$

$$\tilde{R}_{\lambda\rho\mu\nu} = R_{\lambda\rho\mu\nu} = \frac{1}{2} \left( \frac{\partial^2 g_{\lambda\nu}}{\partial x^{\rho} \partial x^{\mu}} + \frac{\partial^2 g_{\rho\mu}}{\partial x^{\lambda} \partial x^{\nu}} - \frac{\partial^2 g_{\lambda\mu}}{\partial x^{\rho} \partial x^{\nu}} - \frac{\partial^2 g_{\rho\nu}}{\partial x^{\lambda} \partial x^{\mu}} \right) + g_{\sigma\omega} \left( \Gamma^{\omega}{}_{\rho\mu} \Gamma^{\sigma}{}_{\lambda\nu} - \Gamma^{\omega}{}_{\rho\nu} \Gamma^{\sigma}{}_{\lambda\mu} \right) .$$

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- General decomposition of the affine connection and the curvature tensor:

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu} ,$$

$$N^{\lambda}{}_{\mu\nu} = \frac{1}{2}\, T^{\lambda}{}_{\mu\nu} - T_{(\mu}{}^{\lambda}{}_{\nu)} + \frac{1}{2}\, Q^{\lambda}{}_{\mu\nu} - Q_{(\mu}{}^{\lambda}{}_{\nu)} ,$$

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|} N^{\lambda}{}_{\rho|\nu]} + 2 N^{\lambda}{}_{\sigma[\mu|} N^{\sigma}{}_{\rho|\nu]} .$$

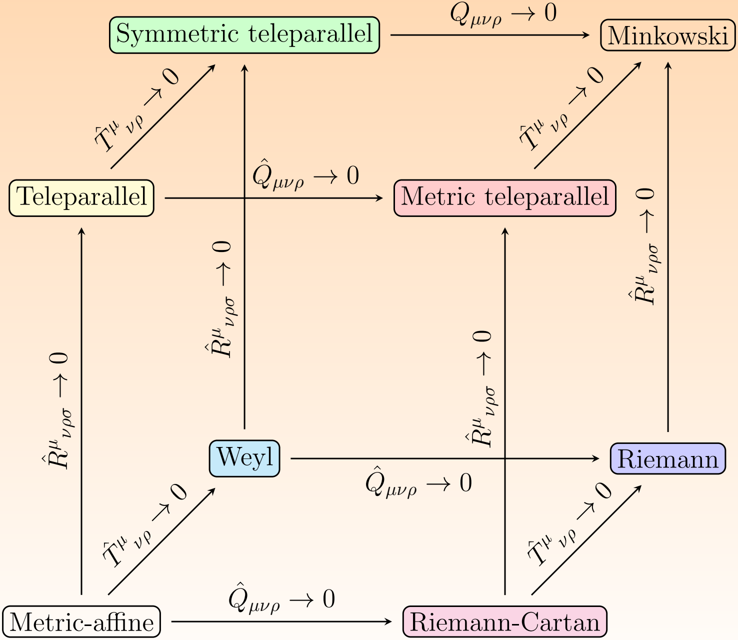


Figure: Classification of metric-affine geometries - Cube

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- Teleparallel condition:

$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho} N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma} N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho} N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma} N^{\tau}{}_{\nu\rho} = 0\,.$$

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**Ricci scalar decomposition**

$$\tilde{R} = R + \left(T + 2\mathring{\nabla}_{\mu}(\sqrt{-g}T^{\rho}{}_{\rho}{}^{\mu})\right) + \left(Q + \mathring{\nabla}_{\mu}Q^{\mu\nu}{}_{\nu} - \mathring{\nabla}_{\nu}Q_{\mu}{}^{\mu\nu}\right) + C = 0$$

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with

$$T := T^{\rho\lambda\kappa} T_{\rho\lambda\kappa} + 2T^{\rho\lambda\kappa} T_{\kappa\rho\lambda} - 4T_{\rho}{}^{\kappa}{}_{\kappa} T^{\rho\lambda}{}_{\lambda}\,, \quad \text{Torsion scalar}\,,$$

$$Q := -\frac{1}{4} Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2} Q_{\alpha\beta\gamma} Q^{\beta\alpha\gamma} + \frac{1}{4} Q_{\alpha} Q^{\alpha} - \frac{1}{2} Q_{\alpha} \bar{Q}^{\alpha}\,, \text{ Nonmetricity scalar}\,,$$

$$C := 2(Q_{\kappa\rho\lambda} T^{\lambda\kappa\rho} + Q_{\rho}{}^{\sigma}{}_{\sigma} T^{\rho\kappa}{}_{\kappa} - Q^{\sigma}{}_{\sigma\rho} T^{\rho\kappa}{}_{\kappa})\,.$$

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- Torsional teleparallel theories (or Metric teleparalellism) assumes $Q = \tilde{R} = 0$ and then $R = -T + B$ and then the TEGR:

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- Then, these two theories are equivalent (classicality) to GR.

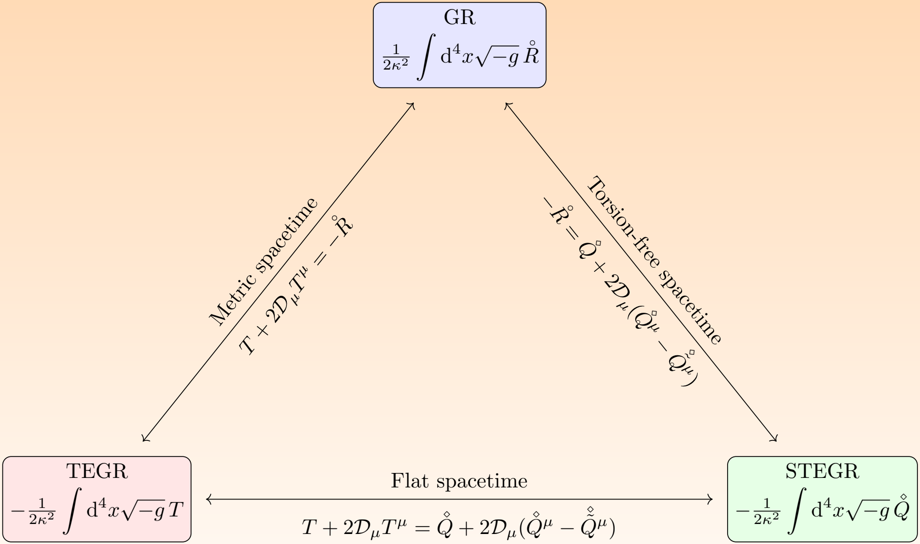


Figure: Geometrical trinity of gravity (S. Bahamonde et.al., “Teleparallel Gravity: From Theory o Cosmology,” Rept. Prog. Phys. **86** (2023) no.2, 026901.; J. Beltrán Jiménez, L. Heisenberg and . S. Koivisto, “The Geometrical Trinity of Gravity,” Universe **5** (2019) no.7, 173.)

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$$\mathcal{L}_{\rm NGR} = c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho}\,.$$

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  2. However, it is strongly coupled in Minkowski and even in non-flat FLRW S. Bahamonde, K. F. Dialektopoulos, M. Hohmann, J. Levi Said, C. Pfeifer and E. N. Saridakis, Eur. Phys. J. C **83** (2023) no.3, 193

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  2. Interesting in black holes: scalarization (spontaneous) is triggered by torsion. S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107** (2023) no.10, 104013; Phys. Rev. D **108** (2023) no.6, 064044

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$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i + G_{\text{Tele}}(\phi, X, T, T_{\text{ax}}, T_{\text{vec}}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})$$

where:

$$\mathring{L}_2 = G_2(\phi, X)\,, \quad \mathring{L}_3 = -G_3(\phi, X)\mathring{\Box}\phi\,,$$

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$$J_6 = t^{\sigma\mu\nu}t_\sigma{}^{\bar{\mu}\bar{\nu}}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar{\mu}}\phi_{;\bar{\nu}}\,, J_8 = t^{\sigma\mu\nu}t_{\sigma\mu}{}^{\bar{\nu}}\phi_{;\nu}\phi_{;\bar{\nu}}\,, \quad J_{10} = \epsilon^\mu{}_{\nu\rho\sigma}a^\nu t^{\alpha\rho\sigma}\phi_{;\mu}\phi_{;\alpha}\,.$$

with $T_\mu = T^\lambda{}_{\mu\lambda}$, $S_\mu = \varepsilon_{\mu\nu\rho\sigma}T^{\nu\sigma\rho}$, $t^\lambda{}_{\mu\nu} = T^\lambda{}_{\mu\nu} - \frac{1}{3}\left(\delta^\lambda{}_\nu T_\mu - \delta^\lambda{}_\mu T_\nu\right) - \frac{1}{6}\varepsilon^\lambda{}_{\rho\mu\nu}S^\rho$.

# Reviving Horndeski using Teleparallel gravity

- As we said before, for $G_{\rm Tele}=0$ (standard case), one gets that to achieve a theory consistent with the GW observations $c_T=1$, one requires $G_5(\phi,X)=\text{constant}$ and $G_4(\phi,X)=G_4(\phi)$. Hence, Horndeski gravity is highly constraint.

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## Teleparallel Lagrangian respecting $c_T = 1$ $(\alpha_T = 0)$

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$$+ G_4(\phi, X)\,(-T + B) + G_{4,X}\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} + 4J_5\right]$$
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# Overview of the Talk

# MAG as a Gauge Theory

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$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac}g_{bd}e^{c\lambda}e^{d\rho}Q_{\mu\lambda\rho}\,,$$
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- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity.

# Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x \sqrt{-g} \Big[ \mathcal{L}_m - \frac{1}{16\pi} \mathcal{L}_g \big( \tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q} \big) \Big] \, .$$

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# Quadratic Poincaré gauge theory - ghost issue

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+ d_1\tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\,d^4x\,.$$

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- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors $T_{\mu}$ and $S_{\mu}$ propagate a ghost.

# Cubic Poincaré gauge theory

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$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St} \,,$$

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\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
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- We showed that by including these Poincaré gauge invariants, ghost issue is solved!
- Further, we showed that nonmetricity can be added dynamically as a ghost-free Gauge theory(S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) no.8, 084058)

# Overview of the Talk

# Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

$$\#24 \to \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \to \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

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**New exact black hole solution with three intrinsic charges**

$$g_{tt} = -\frac{1}{g_{rr}} = \Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(H_1 \kappa_{\rm s}^2 + H_2 \kappa_{\rm d}^2 + H_3 \kappa_{\rm sh}^2\right).$$

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- Here, $\kappa_{\rm s}, \kappa_{\rm d}$ and $\kappa_{\rm sh}$ represent the spin, dilation and shear intrinsic charges.

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- It is important to mention that the in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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4. The intrinsic dilation charge $\kappa_{\rm d}$: refers to a scaling transformation — stretching or shrinking spacetime volume without changing shape.
5. The intrinsic shear charge $\kappa_{\rm d}$: deformations that change the shape of an object without changing its volume.

# Axially symmetric black holes - spin orbit interaction

## Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

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- In this case, this would be an interaction between the angular momentum $a$ and the intrinsic spin $\kappa_s$ in 4D.
- We found a slowly rotating Kerr-like black hole solution in Poincaré Cubic gravity with a dynamical torsion and: (work in progress)

## 4D Axially symmetric black holes - spin orbit interaction

- In 4D: This problem involves the full axially symmetric torsion containing 24 dof and 4 dof of the metric. It is a extremely difficult problem to solve the systems.
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- Is there any interesting new effect that can emerge from this analogy?

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    - Cubic Poincaré: slowly rotating Kerr-like BH with **gravitational spin-orbit interaction**: $\mathcal{L} \propto \frac{1}{16\pi G r^6}\, a d_1 \kappa_s m(\alpha \cos\vartheta + \beta \sin\vartheta)$.