

Theories with Torsion and Nonmetricity: From Black Holes to Cosmology

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Mainly jointly with Jorge Gigante Valcarcel

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Phys. Rev. D **109** (2024) no.10, 10;

Phys. Rev. D **110** (2024) no.12, 12

Phys. Rev. D **111** (2025) no.8, 084058;

arXiv:2506.17017; arXiv:2507.02362; arXiv:2508.20035.



- 1 Metric-Affine gravity and Gauge approach with Cubic interactions
- 2 Black holes with torsion and nonmetricity
 - Spherically symmetric black holes
 - Axially symmetric black holes
- 3 Cosmology with torsion and nonmetricity
- 4 Main results and Possible directions

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How to modify it?

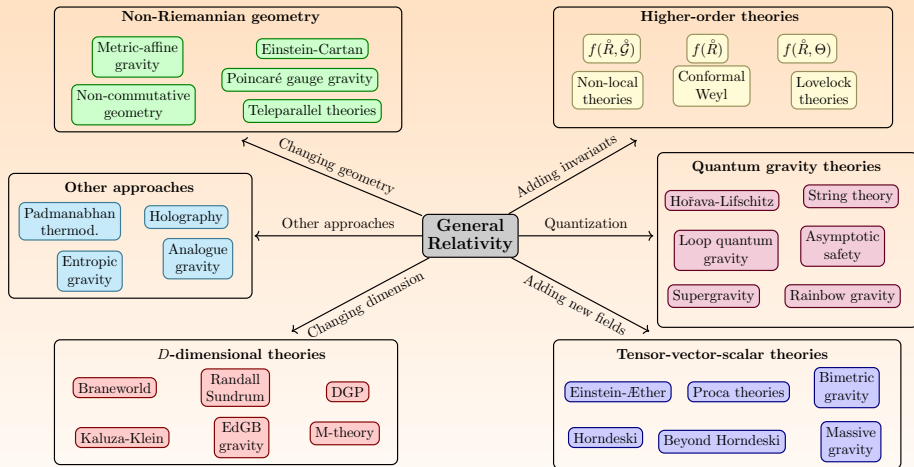


Figure: Classification of theories of gravity. (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

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- The general curvature is defined as in GR but changing $\Gamma^\lambda_{\mu\nu}$ to $\tilde{\Gamma}^\lambda_{\mu\nu}$:

$$\tilde{R}^\lambda_{\rho\mu\nu} = \partial_\mu \tilde{\Gamma}^\lambda_{\rho\nu} - \partial_\nu \tilde{\Gamma}^\lambda_{\rho\mu} + \tilde{\Gamma}^\lambda_{\sigma\mu} \tilde{\Gamma}^\sigma_{\rho\nu} - \tilde{\Gamma}^\lambda_{\sigma\nu} \tilde{\Gamma}^\sigma_{\rho\mu}$$

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

General descriptions

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- Recall that one can "go back" to the standard curvature by contracting this quantity with tetrads:

$$\tilde{R}^\lambda{}_{\rho\mu\nu} = e_a{}^\lambda e_b{}^\rho F^a{}_{b\mu\nu}$$

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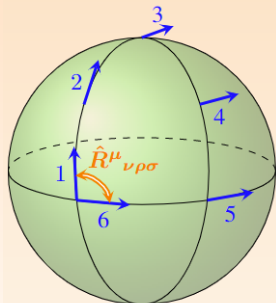
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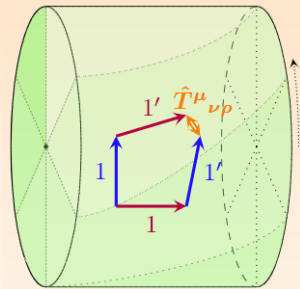
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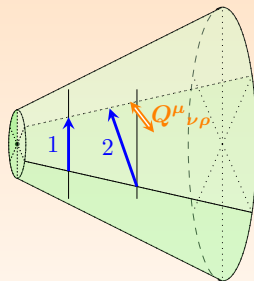
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- Unique scalar and pseudoscalar curvatures:

$$\tilde{R} = g^{\mu\nu} \tilde{R}_{\mu\nu}, \quad * \tilde{R} = \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu}.$$

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu},$$

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- General decomposition of the the curvature tensor:

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$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac} \omega^c{}_{b\mu} - g_{bc} \omega^c{}_{a\mu} = g_{ac} g_{bd} e^{c\lambda} e^{d\rho} Q_{\mu\lambda\rho},$$

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- When nonmetricity is vanishing, the group becomes the Poincaré group \implies Poincaré gauge theories of gravity (PG).

Teleparallel theories: Trinity of Gravity

- Let us consider another particular interesting case known as "Teleparallel Geometries" with the following condition:

$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho}N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma}N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho}N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma}N^{\tau}{}_{\nu\rho} = 0.$$

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Ricci scalar decomposition

$$\tilde{R} = R + \left(T + 2\overset{\circ}{\nabla}_{\mu}(\sqrt{-g}T^{\rho}{}_{\rho}{}^{\mu}) \right) + \left(Q + \overset{\circ}{\nabla}_{\mu}Q^{\mu\nu}{}_{\nu} - \overset{\circ}{\nabla}_{\nu}Q_{\mu}{}^{\mu\nu} \right) + C = 0$$

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$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho}N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma}N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho}N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma}N^{\tau}{}_{\nu\rho} = 0.$$

- Now, by contracting the curvature tensor $\tilde{R} = g^{\mu\nu}\tilde{R}^{\rho}{}_{\mu\rho\nu}$ we find

Ricci scalar decomposition

$$\tilde{R} = R + \left(T + 2\overset{\circ}{\nabla}_{\mu}(\sqrt{-g}T^{\rho}{}_{\rho}{}^{\mu})\right) + \left(Q + \overset{\circ}{\nabla}_{\mu}Q^{\mu\nu}{}_{\nu} - \overset{\circ}{\nabla}_{\nu}Q_{\mu}{}^{\mu\nu}\right) + C = 0$$

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with

$$T := T^{\rho\lambda\kappa}T_{\rho\lambda\kappa} + 2T^{\rho\lambda\kappa}T_{\kappa\rho\lambda} - 4T_\rho{}^\kappa{}_\kappa T^{\rho\lambda}{}_\lambda, \quad \text{Torsion scalar},$$

$$Q := -\frac{1}{4} Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2} Q_{\alpha\beta\gamma} Q^{\beta\alpha\gamma} + \frac{1}{4} Q_\alpha Q^\alpha - \frac{1}{2} Q_\alpha \bar{Q}^\alpha, \quad \text{Nonmetricity scalar},$$

$$C := 2(Q_{\kappa\rho\lambda} T^{\lambda\kappa\rho} + Q_\rho{}^\sigma T^{\rho\kappa}{}_\kappa - Q^\sigma{}_\sigma T^{\rho\kappa}{}_\kappa).$$

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- Torsional teleparallel theories (or Metric teleparallelism) assumes $Q = \tilde{R} = 0$ and then $R = -T + B$ and then the TEGR:

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- Then, these two theories are equivalent (classicality) to GR.

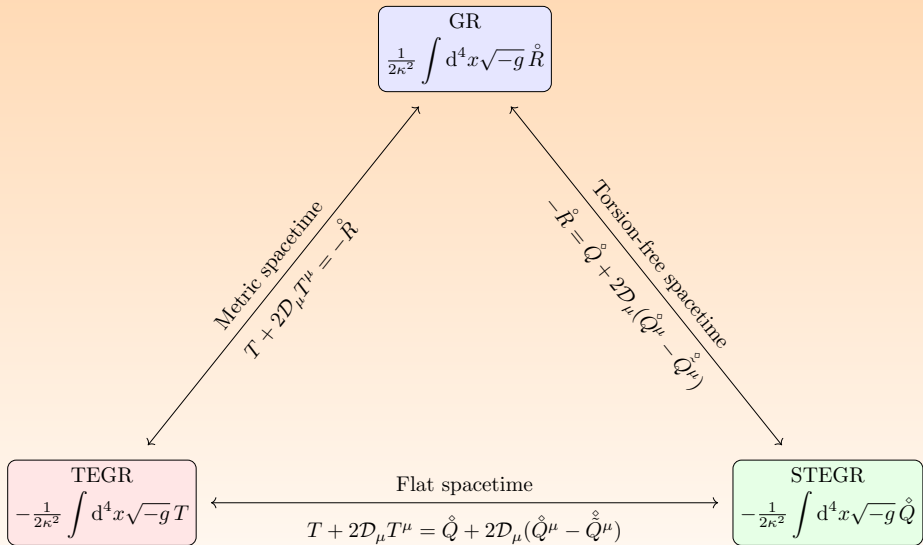


Figure: Geometrical trinity of gravity (S. Bahamonde et.al., “Teleparallel Gravity: From Theory to Cosmology,” Rept. Prog. Phys. **86** (2023) no.2, 026901.; J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, “The Geometrical Trinity of Gravity,” Universe **5** (2019) no.7, 173.)

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x \sqrt{-g} \left[\mathcal{L}_m - \frac{1}{16\pi} \mathcal{L}_g(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q}) \right].$$

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Quadratic Poincaré gauge theory - ghost issue

- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi} \int \left[-R + c_2 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2} (2c_1 + c_2) \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\mu\nu\lambda\rho} + c_1 \tilde{R}_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho\mu\nu} \right. \\ \left. + d_1 \tilde{R}_{\mu\nu} (\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}) + \frac{1}{2} (m_T^2 T_\mu T^\mu + m_S^2 S_\mu S^\mu + m_t^2 t_{\lambda\mu\nu} t^{\lambda\mu\nu}) \right] \sqrt{-g} d^4x .$$

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- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_μ and S_μ propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel,

Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}_{\text{curv-tors}}^{(3)} = \mathcal{L}_{\tilde{R}TT}^{(3)} + \mathcal{L}_{\tilde{R}SS}^{(3)} + \mathcal{L}_{\tilde{R}tt}^{(3)} + \mathcal{L}_{\tilde{R}TS}^{(3)} + \mathcal{L}_{\tilde{R}Tt}^{(3)} + \mathcal{L}_{\tilde{R}St}^{(3)},$$

$$\mathcal{L}_{\tilde{R}TT}^{(3)} = h_1 \tilde{R}_{\mu\nu} T^\mu T^\nu + h_2 \tilde{R} T_\mu T^\mu, \quad \mathcal{L}_{\tilde{R}SS}^{(3)} = h_3 \tilde{R}_{\mu\nu} S^\mu S^\nu + h_4 \tilde{R} S_\mu S^\mu,$$

$$\begin{aligned} \mathcal{L}_{\tilde{R}tt}^{(3)} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_\sigma^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho} t^{\sigma\mu\nu} \\ &+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}^{\lambda} t^{\rho\mu\nu} \\ &+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu}^{\lambda} t^{\rho\mu}, \end{aligned}$$

$$\mathcal{L}_{\tilde{R}TS}^{(3)} = h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_\sigma S^\sigma + h_{14} \varepsilon_{\nu}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^\mu S^\nu + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_\mu S_\nu,$$

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$$\begin{aligned} \mathcal{L}_{\tilde{R}St}^{(3)} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau^{\rho\mu\nu} S^\gamma t^{\alpha\tau}{}_\gamma + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_\tau^{\rho\mu\nu} S^\gamma t_\gamma{}^{\alpha\tau} \\ &+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^\rho{}_{\mu\tau\nu} S^\gamma t_\gamma{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\gamma\rho\tau} \\ &+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^\alpha t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^\mu{}_\gamma S^\alpha t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_\sigma t^{\sigma\mu\nu}. \end{aligned}$$

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- We showed that by including these Poincaré gauge invariants, ghost issue in the vector sector is solved!

Cubic Metric-Affine gauge theory

- Now let us include nonmetricity and decompose the fields as:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3} \left(\delta^{\lambda}{}_{\nu} T_{\mu} - \delta^{\lambda}{}_{\mu} T_{\nu} \right) + \frac{1}{6} \varepsilon^{\lambda}{}_{\rho\mu\nu} S^{\rho} + t^{\lambda}{}_{\mu\nu} ,$$

$$Q_{\lambda\mu\nu} = g_{\mu\nu} W_{\lambda} + \frac{1}{2} \left(g_{\lambda\mu} \Lambda_{\nu} + g_{\lambda\nu} \Lambda_{\mu} \right) - \frac{1}{4} g_{\mu\nu} \Lambda_{\lambda} + \frac{1}{3} \varepsilon_{\lambda\rho\sigma(\mu} \Omega_{\nu)}{}^{\rho\sigma} + q_{\lambda\mu\nu} .$$

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- From torsion: 1 vector T_μ and 1 axial vector S_μ

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- We showed that by introducing these terms that are part of the Gauge approach, one can eliminate all known vector/axial ghosts.

1 Metric-Affine gravity and Gauge approach with Cubic interactions

2 **Black holes with torsion and nonmetricity**

- Spherically symmetric black holes
- Axially symmetric black holes

3 Cosmology with torsion and nonmetricity

4 Main results and Possible directions

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0.$$

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- Static and spherically symmetric space-times:

$$\#10 \rightarrow \#2 \left\{ ds^2 = \Psi_1(r) dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) ; \right.$$

$$\#24 \rightarrow \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

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$$g_{tt} = -\frac{1}{g_{rr}} = \Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2} \left(H_1 \kappa_s^2 + H_2 \kappa_d^2 + H_3 \kappa_{sh}^2 \right).$$

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- All the masses of the tensor modes of torsion and nonmetricity are different from zero \implies We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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Relationship Riemannian Gauss-Bonnet and Teleparallel Gauss-Bonnet invariants

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- Then, two Teleparallel Gauss-Bonnet invariants appear in the Teleparallel framework. T_G is a topological invariant in $4D$ and B_G is a boundary term (in all dimensions).

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- We have found hairy black hole solutions with spontaneous scalarization process which are sourced by torsion in two papers (S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107** (2023) no.10, 104013; Phys. Rev. D **108** (2023) no.6, 064044).

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- Teleparallel offers a new way for studying BH endowed with hairs that can have different properties as in the Riemannian sector.

Electrodynamics coupled with torsion

- Now, consider another theory in Riemann-Cartan geometry with couplings between the electromagnetic field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $\tilde{R}^\lambda{}_{\rho\mu\nu}$: (S. Bahamonde, J. Maggiolo and C. Pfeifer, arXiv:2507.02362)

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- The Maxwell Eq. has a torsion source: $2k_1 \nabla_\mu F^{\mu\nu} = k_3 \nabla_\mu \tilde{R}^{[\mu\nu]}$.
- We found the following black hole solution:

$$t_1(r) = \frac{\Psi'(r)}{2\Psi(r)} + \frac{c_1}{\Psi(r)} + \frac{\kappa_s}{r\Psi(r)}, \quad t_2(r) = \pm\Psi(r) \left(t_4(r) - t_1(r) - \frac{A_0''(r)}{A_0'(r)} \right),$$
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- The metric and electric potential behave as

$$\Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2} \left(k_1 q^2 - \frac{1}{2} k_3 \kappa_s q - \frac{1}{32k_2} k_3^2 q^2 \right), \quad A_\mu = \left(\frac{q}{r}, 0, 0, 0 \right).$$

Electrodynamics coupled with torsion

- Now, consider another theory in Riemann-Cartan geometry with couplings between the electromagnetic field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $\tilde{R}^\lambda{}_{\rho\mu\nu}$: (S. Bahamonde, J. Maggiolo and C. Pfeifer, arXiv:2507.02362)

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- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

Spin-Orbit Interaction in Atomic Physics

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

Axially symmetric black holes - spin orbit interaction

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- We initially focused on the search of black hole solutions that at least can display such an interaction in the gravitational action, regardless if it does not provide any modification in the metric tensor.
- We focused on a degenerate model of cubic PG theory, which provides static and spherically symmetric solutions with a spin charge that does not affect the Schwarzschild geometry

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Lagrangian like a gravitational spin-orbit interaction in the solution

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- If $G(r, \vartheta) = \frac{1}{\sqrt{-g}} \frac{\Psi'(r)}{r} \cos \vartheta$, \mathcal{L}_{SOI} this term would provide a term analogous to the **Thomas precession of atomic systems**, but in this case with a **purely gravitational origin**.

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- The form of torsion contains 24 dof being non-zero.
- Is it possible to find a solution in the non-degenerate theory and find a modified Kerr metric with interactions between κ_s and a in the metric?

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- The arbitrary function $G(r, \vartheta)$ is encoded solely in the axial mode of the torsion tensor of the solution.
- Therefore, trajectories of Dirac particles minimally coupled to torsion will accordingly experience deviations from the geodesic motion, which can already be measured in the semiclassical limit by an acceleration of the form:

$$u^\lambda \nabla_\lambda u_\mu = \frac{1}{4m_s} \hat{R}_{\lambda\rho\mu\nu} \bar{b}_0 \sigma^{\lambda\rho} b_0 u^\nu ,$$

where u_μ represents the four-velocity of the particle, b_0 its normalised state, m_s its mass, $\sigma^{\lambda\rho}$ the spin matrices and $\hat{R}_{\lambda\rho\mu\nu}$ the part of the Riemann-Cartan curvature tensor that includes corrections from the axial mode alone.

- 1 Metric-Affine gravity and Gauge approach with Cubic interactions
- 2 Black holes with torsion and nonmetricity
 - Spherically symmetric black holes
 - Axially symmetric black holes
- 3 Cosmology with torsion and nonmetricity
- 4 Main results and Possible directions

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- The majority of these studies predominantly focus on torsion.
- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0.$$

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}\bar{g} &= -\bar{n}_\mu \bar{n}_\nu dx^\mu \otimes dx^\nu + \bar{P}_{\mu\nu} dx^\mu \otimes dx^\nu = -N^2 dt \otimes dt + a^2 \gamma_{ij} dx^i \otimes dx^j, \\ \bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t) \bar{n}_{[\mu} \bar{P}_{\nu]}{}^\lambda + 2T_2(t) \bar{\varepsilon}^\lambda{}_{\mu\nu\rho} \bar{n}^\rho, \\ \bar{Q}^\lambda{}_{\mu\nu} &= 2Q_1(t) \bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t) \bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t) \bar{P}_{\lambda(\mu} \bar{n}_{\nu)},\end{aligned}$$

where $\gamma_{ij} dx^i \otimes dx^j = \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3} \Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4} \Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho ,$$

which contains 5 different sources dof related to the intrinsic spin, dilations, and shears.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_{\mu}T^{\mu}_{\alpha} - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_{\mu}\tilde{\nabla}_{\nu}(\sqrt{-g}\Delta_{\alpha}^{\mu\nu}) + 2T_{\mu\alpha}{}^{\lambda}\tilde{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}^{\mu\nu}) = 0.$$

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- In our Cubic Poincaré Gauge Gravity theory, we have the modified FLRW equations of the form: S. Bahamonde, R. Briffa, K. Dialektopoulos, D. Iosifidis and J. Levi Said,

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$$3H^2 + f(T_1(t), T_2(t)) = \kappa^2 \rho, \quad 3H^2 + 2\dot{H} + g(T_1(t), T_2(t)) = -\kappa^2 p.$$

and the connection equations:

$$\begin{aligned} 2\kappa^2 {}^{(s)}\Delta_3 &= 3T_1 \left[6h_1(H - T_1)(H - 2T_1) - 6(h_1 - 16h_{13})T_2^2 + m_T^2 \right], \\ \kappa^2 {}^{(s)}\Delta_5 &= 3T_2 \left[48h_{13}(-H^2 + T_1^2 + 2T_2^2) - 3h_1T_1^2 + 8m_S^2 \right]. \end{aligned}$$

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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T .

- The conservation equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 3^{(s)}\Delta_5 (HT_2 + \dot{T}_2) - 3^{(s)}\Delta_3 (\dot{H} - HT_1 + H^2 - \dot{T}_1) .$$

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- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.

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where C_i depends on the theory parameters and torsion constants K_1, K_2 .

- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.
- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

SVT decomposition around FLRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(T)}, h_i^{(T)}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(TT)}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(T)}, B_i^{(T)}, A_i^{(T)}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(T)}, \mathcal{B}_i^{(T)}, \mathcal{A}_i^{(T)}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(TT)}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(TT)}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FLRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(T)}_i, Y^{(T)}_i, Z^{(T)}_i, \kappa^{(T)}_i, Q^{(T)}_i, W^{(T)}_i, C^{(T)}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(T)}_i, \mathcal{Q}^{(T)}_i\}$	2 dof each	4
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- Since we have spin-3, spin-2, spin-1, spin-0 being dynamical, different effects might emerge!

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- 2 Black holes with torsion and nonmetricity
 - Spherically symmetric black holes
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- Are there any interesting new effects that can emerge from this geometrical picture?