Theories with Torsion and Nonmetricity: From Black Holes to Cosmology

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Mainly jointly with Jorge Gigante Valcarcel

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Phys. Rev. D **109** (2024) no.10, 10;
Phys. Rev. D **110** (2024) no.12, 12
Phys. Rev. D **111** (2025) no.8, 084058;
arXiv:2506.17017; arXiv:2507.02362;arXiv:2508.20035.

Outline

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How to modify it?

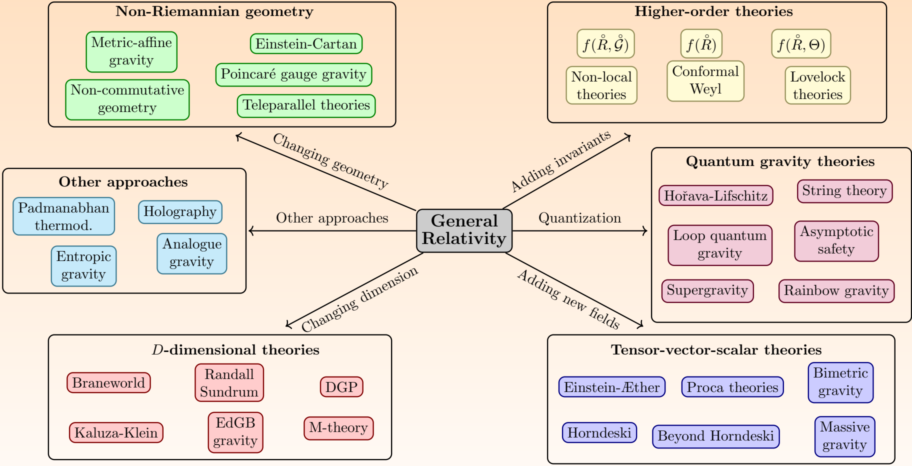

Figure: Classification of theories of gravity. (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

General descriptions

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- In these new variables, one can easily define spinors (as in GR) but one of the main advantage is that the curvature form DOES NOT depend on the tetrad field (or metric):

$$F^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu} \, \omega^c{}_{b\,\nu} - \omega^a{}_{c\nu} \, \omega^c{}_{b\mu} \, .$$

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- Recall that one can "go back" to the standard curvature by contracting this quantity with tetrads:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = e_{a}{}^{\lambda} e^{b}{}_{\rho}\, F^{a}{}_{b\mu\nu}$$

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- In particular, it measures the change of vector components on parallel transport along an infinitesimal closed curve:

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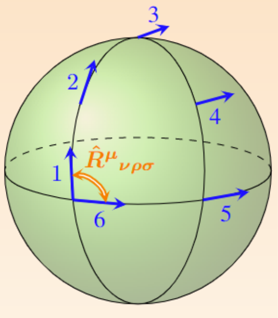

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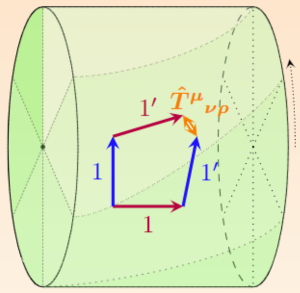

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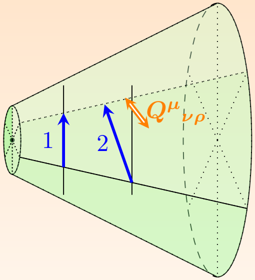

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- Three independent rank-2 tensors defined from the first contractions of the curvature tensor:

$$\tilde{R}_{\mu\nu} = \tilde{R}^{\lambda}{}_{\mu\lambda\nu}\,,\quad \hat{R}_{\mu\nu} = \tilde{R}_{\mu}{}^{\lambda}{}_{\nu\lambda}\,,\quad \tilde{R}^{\lambda}{}_{\lambda\mu\nu} = \nabla_{[\nu}Q_{\mu]\lambda}{}^{\lambda}\,.$$

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- Unique scalar and pseudoscalar curvatures:

$$\tilde{R} = g^{\mu\nu}\tilde{R}_{\mu\nu}\,, \quad *\tilde{R} = \varepsilon^{\lambda\rho\mu\nu}\tilde{R}_{\lambda\rho\mu\nu}\,.$$

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}\,,$$

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- General decomposition of the the curvature tensor:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|}N^{\lambda}{}_{\rho|\nu]} + 2N^{\lambda}{}_{\sigma[\mu|}N^{\sigma}{}_{\rho|\nu]}\,.$$

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$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac}g_{bd}e^{c\lambda}e^{d\rho}Q_{\mu\lambda\rho}\,,$$
$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\,e^b{}_\nu - \omega^a{}_{b\nu}\,e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,$$
$$F^a{}_{b\mu\nu} = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\,\omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\,\omega^c{}_{b\mu} = g_{bc}\,e^a{}_\lambda e^{c\rho}\tilde{R}^\lambda{}_{\rho\mu\nu}\,.$$

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- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity (PG).

Teleparallel theories: Trinity of Gravity

- Let us consider another particular interesting case known as "Teleparallel Geometries" with the following condition:

$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho} N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma} N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho} N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma} N^{\tau}{}_{\nu\rho} = 0\,.$$

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Ricci scalar decomposition

$$\tilde{R} = R + \left(T + 2\mathring{\nabla}_{\mu}(\sqrt{-g}T^{\rho}{}_{\rho}{}^{\mu})\right) + \left(Q + \mathring{\nabla}_{\mu}Q^{\mu\nu}{}_{\nu} - \mathring{\nabla}_{\nu}Q_{\mu}{}^{\mu\nu}\right) + C = 0$$

Teleparallel theories: Trinity of Gravity

- Let us consider another particular interesting case known as ”Teleparallel Geometries” with the following condition:

$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho} N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma} N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho} N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma} N^{\tau}{}_{\nu\rho} = 0\,.$$

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with

$$T := T^{\rho\lambda\kappa}T_{\rho\lambda\kappa} + 2T^{\rho\lambda\kappa}T_{\kappa\rho\lambda} - 4T_{\rho}{}^{\kappa}{}_{\kappa}T^{\rho\lambda}{}_{\lambda}\,, \quad \text{Torsion scalar}\,,$$

$$Q := -\frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} + \frac{1}{2}Q_{\alpha\beta\gamma}Q^{\beta\alpha\gamma} + \frac{1}{4}Q_{\alpha}Q^{\alpha} - \frac{1}{2}Q_{\alpha}\bar{Q}^{\alpha}\,, \ \text{Nonmetricity scalar}\,,$$

$$C := 2(Q_{\kappa\rho\lambda}T^{\lambda\kappa\rho} + Q_{\rho}{}^{\sigma}{}_{\sigma}T^{\rho\kappa}{}_{\kappa} - Q^{\sigma}{}_{\sigma\rho}T^{\rho\kappa}{}_{\kappa})\,.$$

Teleparallel theories: Trinity of Gravity

- Torsional teleparallel theories (or Metric teleparalellism) assumes $Q = \tilde{R} = 0$ and then $R = -T + B$ and then the TEGR:

$$S_{\rm TEGR} = \int \left[-\frac{1}{2\kappa^2} T + L_{\rm m} \right] e \, d^4x \, .$$

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- Then, these two theories are equivalent (classicality) to GR.

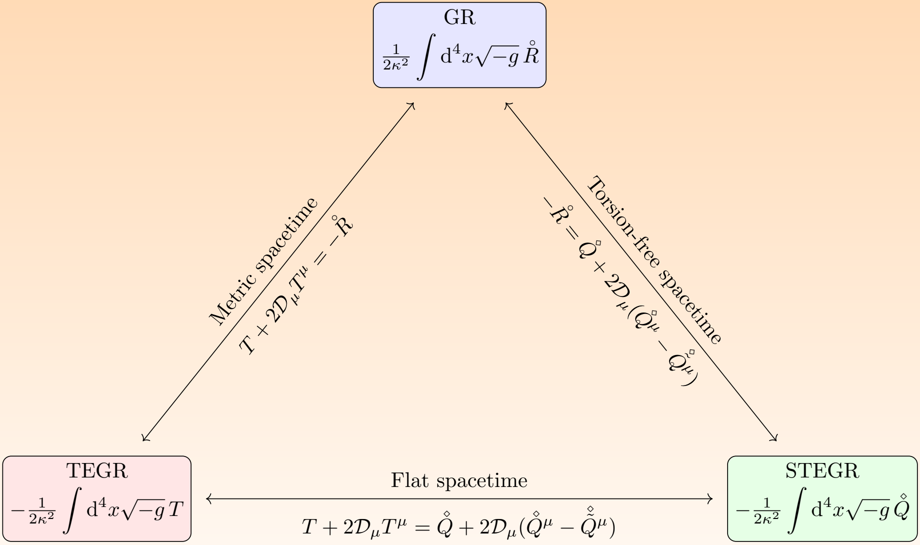

Figure: Geometrical trinity of gravity (S. Bahamonde et.al., "Teleparallel Gravity: From Theory to Cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.; J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, "The Geometrical Trinity of Gravity," Universe **5** (2019) no.7, 173.)

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x\sqrt{-g}\Big[\mathcal{L}_m - \frac{1}{16\pi}\mathcal{L}_g\big(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q}\big)\Big]\,.$$

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$$\begin{aligned}\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu\,,\\ \frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta \omega^a{}_{b\nu}} &= 16\pi\Delta_a{}^{b\nu}\,.\end{aligned}$$

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- Hypermomentum can be split into three parts:

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3. **Intrinsic Shears** term ${}^{(sh)}\Delta_{(\mu\nu)\lambda}$: source of **traceless nonmetricity**

- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
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- Convenient to decompose torsion as

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- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_μ and S_μ propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\text{curv}-\text{tors}} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1\tilde{R}_{\mu\nu}T^{\mu}T^{\nu} + h_2\tilde{R}T_{\mu}T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3\tilde{R}_{\mu\nu}S^{\mu}S^{\nu} + h_4\tilde{R}S_{\mu}S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5\tilde{R}_{\lambda\rho\mu\nu}t_{\sigma}{}^{\lambda\rho}t^{\sigma\mu\nu} + h_6\tilde{R}_{\lambda\rho\mu\nu}t_{\sigma}{}^{\lambda\mu}t^{\sigma\rho\nu} + h_7\tilde{R}_{\lambda\rho\mu\nu}t^{\lambda\rho}{}_{\sigma}t^{\sigma\mu\nu}\\
&\quad + h_8\tilde{R}_{\lambda\rho\mu\nu}t^{\lambda\mu}{}_{\sigma}t^{\sigma\rho\nu} + h_9\tilde{R}_{\lambda\rho\mu\nu}t^{\lambda\mu}{}_{\sigma}t^{\rho\nu\sigma} + h_{10}\tilde{R}_{\lambda\rho}t_{\mu\nu}{}^{\lambda}t^{\rho\mu\nu}\\
&\quad + h_{11}\tilde{R}_{\lambda\rho}t_{\mu\nu}{}^{\lambda}t^{\mu\nu\rho} + h_{12}\tilde{R}t_{\lambda\rho\mu}t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13}\varepsilon^{\lambda\rho\mu\nu}\tilde{R}_{\lambda\rho\mu\nu}T_{\sigma}S^{\sigma} + h_{14}\varepsilon_{\nu}{}^{\lambda\rho\sigma}\tilde{R}_{\lambda\rho\mu\sigma}T^{\mu}S^{\nu} + h_{15}\varepsilon^{\lambda\rho\mu\nu}\tilde{R}_{\lambda\rho}T_{\mu}S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16}\tilde{R}_{\lambda\rho\mu\nu}T^{\nu}t^{\lambda\rho\mu} + h_{17}\tilde{R}_{\lambda\rho\mu\nu}T^{\rho}t^{\lambda\mu\nu} + h_{18}\tilde{R}_{\lambda\rho}T_{\mu}t^{\mu\lambda\rho} + h_{19}\tilde{R}_{\lambda\rho}T_{\mu}t^{\lambda\rho\mu},\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20}\varepsilon_{\alpha\rho\mu\nu}\tilde{R}_{\tau}{}^{\rho\mu\nu}S^{\gamma}t^{\alpha\tau}{}_{\gamma} + h_{21}\varepsilon_{\alpha\rho\mu\nu}\tilde{R}_{\tau}{}^{\rho\mu\nu}S^{\gamma}t_{\gamma}{}^{\alpha\tau}\\
&\quad + h_{22}\varepsilon_{\alpha\rho}{}^{\mu\nu}\tilde{R}^{\rho}{}_{\mu\tau\nu}S^{\gamma}t_{\gamma}{}^{\alpha\tau} + h_{23}\varepsilon_{\alpha\rho}{}^{\mu\nu}\tilde{R}_{\gamma\mu\tau\nu}S^{\alpha}t^{\gamma\rho\tau}\\
&\quad + h_{24}\varepsilon_{\alpha\rho}{}^{\mu\nu}\tilde{R}_{\gamma\mu\tau\nu}S^{\alpha}t^{\rho\tau\gamma} + h_{25}\varepsilon_{\alpha\rho\tau\mu}\tilde{R}^{\mu}{}_{\gamma}S^{\alpha}t^{\rho\tau\gamma} + h_{26}\varepsilon_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho}S_{\sigma}t^{\sigma\mu\nu}\,.
\end{aligned}$$

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\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t^{\alpha\tau}{}_{\gamma} + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau}\\
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\end{aligned}$$

- We showed that by including these Poincaré gauge invariants, ghost issue in the vector sector is solved!

Cubic Metric-Affine gauge theory

- Now let us include nonmetricity and decompose the fields as:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,,$$

$$Q_{\lambda\mu\nu} = g_{\mu\nu}W_{\lambda} + \frac{1}{2}\left(g_{\lambda\mu}\Lambda_{\nu} + g_{\lambda\nu}\Lambda_{\mu}\right) - \frac{1}{4}g_{\mu\nu}\Lambda_{\lambda} + \frac{1}{3}\varepsilon_{\lambda\rho\sigma(\mu}\Omega_{\nu)}{}^{\rho\sigma} + q_{\lambda\mu\nu}\,.$$

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Cubic Metric-Affine gravity Lagrangian

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- We showed that by introducing these terms that are part of the Gauge approach, one can eliminate all known vector/axial ghosts.

Outline

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

$$\#24 \to \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \to \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

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New exact black hole solution with three intrinsic charges

$$g_{tt} = -\frac{1}{g_{rr}} = \Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\Big(H_1\kappa_{\rm s}^2 + H_2\kappa_{\rm d}^2 + H_3\kappa_{\rm sh}^2\Big)\,.$$

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- It is important to mention that the in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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5. The intrinsic shear charge $\kappa_{\rm sh}$: deformations that change the shape of an object without changing its volume.

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Relationship Riemannian Gauss-Bonnet and Teleparallel Gauss-Bonnet invariants

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Here, D_λ is the cov derivative of the general connection.

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- Then, two Teleparallel Gauss-Bonnet invariants appear in the Teleparallel framework. T_G is a topological invariant in $4D$ and B_G is a boundary term (in all dimensions).

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and a potential

$$U(r) = \left(1 - \frac{2M}{r}\right)\Big[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \frac{32M}{r^5\beta}\alpha_3\ddot{\mathcal{G}}_3(\psi_0) + \frac{48M^2}{r^6\beta}(\alpha_3\ddot{\mathcal{G}}_3(\psi_0) + \alpha_2\ddot{\mathcal{G}}_2(\psi_0))\Big]\,.$$

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- We have found hairy black hole solutions with spontaneous scalarization process which are sourced by torsion in two papers (S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107** (2023) no.10, 104013; Phys. Rev. D **108** (2023) no.6, 064044).

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- We were also able to find black holes with a strong scalar field close to the horizon but with a vanishing scalar charge.
- Teleparallel offers a new way for studying BH endowed with hairs that can have different properties as in the Riemannian sector.

Electrodynamics coupled with torsion

- Now, consider another theory in Riemann-Cartan geometry with couplings between the electromagnetic field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $\tilde{R}^{\lambda}{}_{\rho\mu\nu}$: (S. Bahamonde, J. Maggiolo and C. Pfeifer, arXiv:2507.02362)

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$$\Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(k_1 q^2 - \frac{1}{2}k_3\kappa_s q - \frac{1}{32k_2}k_3^2 q^2\right)\,,\quad A_\mu = \left(\frac{q}{r},0,0,0\right).$$

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- New coupling between intrinsic spin charge κ_s and electric charge q.
- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

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- We initially focused on the search of black hole solutions that at least can display such an interaction in the gravitational action, regardless if it does not provide any modification in the metric tensor.
- We focused on a degenerate model of cubic PG theory, which provides static and spherically symmetric solutions with a spin charge that does not affect the Schwarzschild geometry

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Lagrangian like a gravitational spin-orbit interaction in the solution

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- The form of torsion contains 24 dof being non-zero.
- Is it possible to find a solution in the non-degenerate theory and find a modified Kerr metric with interactions between κ_s and a in the metric?

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- The arbitrary function $G(r,\vartheta)$ is encoded solely in the axial mode of the torsion tensor of the solution.
- Therefore, trajectories of Dirac particles minimally coupled to torsion will accordingly experience deviations from the geodesic motion, which can already be measured in the semiclassical limit by an acceleration of the form:

$$u^{\lambda}\nabla_{\lambda}u_{\mu}=\frac{1}{4m_{\rm s}}\hat{R}_{\lambda\rho\mu\nu}\bar{b}_0\sigma^{\lambda\rho}b_0u^{\nu}\,,$$

where u_μ represents the four-velocity of the particle, b_0 its normalised state, $m_{\rm s}$ its mass, $\sigma^{\lambda\rho}$ the spin matrices and $\hat{R}_{\lambda\rho\mu\nu}$ the part of the Riemann-Cartan curvature tensor that includes corrections from the axial mode alone.

Outline

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- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0\,.$$

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2 \gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\,\bar{n}_{[\mu}\bar{P}_{\nu]}{}^\lambda + 2T_2(t)\,\bar{\varepsilon}^{\lambda}{}_{\mu\nu\rho}\bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t)\bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t)\bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t)\bar{P}_{\lambda(\mu}\bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2 \mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

Cosmology in MAG - matter

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3}\Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4}\Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho\,,$$

which contains 5 different sources dof related to the intrinsic spin, dilations, and shears.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_\mu T^\mu{}_\alpha - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_\mu\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\alpha{}^{\mu\nu}) + 2T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) = 0\,.$$

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$$3H^2 + f(T_1(t),T_2(t)) \;=\; \kappa^2\rho\,, \quad 3H^2 + 2\dot{H} + g(T_1(t),T_2(t)) = -\kappa^2 p\,.$$

and the connection equations:

$$\begin{aligned}
2\kappa^2\,{}^{(\mathrm{s})}\Delta_3 &= 3T_1\Big[6h_1(H-T_1)(H-2T_1) - 6(h_1-16h_{13})T_2^2 + m_T^2\Big]\,,\\
\kappa^2\,{}^{(\mathrm{s})}\Delta_5 &= 3T_2\Big[48h_{13}\left(-H^2+T_1^2+2T_2^2\right) - 3h_1T_1^2 + 8m_S^2\Big]\,.
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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T.

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- Then the first FLRW equation can be written as

$$\left(C_3+\frac{C_1}{a^2}\right)H^2=\frac{\rho_{\mathrm{m0}}}{a^3}+\frac{(\rho_{\mathrm{r0}}-C_4)}{a^4}+\frac{\rho_{\mathrm{de0}}}{a^{3(w+1)}}-\frac{C_2}{a^2}$$

where C_i depends on the theory parameters and torsion constants K_1, K_2.

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- In this case, the connection equations are fully determined and the system decouples.
- Then the first FLRW equation can be written as

$$\left(C_3 + \frac{C_1}{a^2}\right)H^2 = \frac{\rho_{\mathrm{m0}}}{a^3} + \frac{(\rho_{\mathrm{r0}} - C_4)}{a^4} + \frac{\rho_{\mathrm{de0}}}{a^{3(w+1)}} - \frac{C_2}{a^2}$$

  where C_i depends on the theory parameters and torsion constants K_1, K_2.
- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.

- The conservation equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 3\,{}^{(\mathrm{s})}\Delta_5\left(HT_2 + \dot{T}_2\right) - 3\,{}^{(\mathrm{s})}\Delta_3\left(\dot{H} - HT_1 + H^2 - \dot{T}_1\right) .$$

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- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

SVT decomposition around FLRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(\mathrm{T})}, h_i^{(\mathrm{T})}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(\mathrm{TT})}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(\mathrm{T})}, B_i^{(\mathrm{T})}, A_i^{(\mathrm{T})}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(\mathrm{T})}, \mathcal{B}_i^{(\mathrm{T})}, \mathcal{A}_i^{(\mathrm{T})}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(\mathrm{TT})}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(\mathrm{TT})}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, “Cosmological Perturbation Theory in Metric-Affine Gravity,” Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FLRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\mathrm{T})}{}_i, Y^{(\mathrm{T})}{}_i, Z^{(\mathrm{T})}{}_i, \kappa^{(\mathrm{T})}{}_i, Q^{(\mathrm{T})}{}_i, W^{(\mathrm{T})}{}_i, C^{(\mathrm{T})}{}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(\mathrm{T})}{}_i, \mathcal{Q}^{(\mathrm{T})}{}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(\mathrm{TT})}{}_{ij}, Q^{(\mathrm{TT})}{}_{ij}, C^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	6
1 rank-2 pseudotensor	$\{\mathcal{Q}^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	2
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Table: Perturbation spectrum for the nonmetricity tensor.

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- Since we have spin-3,spin-2,spin-1,spin-0 being dynamical, different effects might emerge!

Outline

1. Metric-Affine gravity and Gauge approach with Cubic interactions
2. Black holes with torsion and nonmetricity
   - Spherically symmetric black holes
   - Axially symmetric black holes
3. Cosmology with torsion and nonmetricity
4. Main results and Possible directions

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- Developed **cosmological perturbation theory** including helicity-3 sector of spin-3 nonmetricity.
- We formulated the algebraic classification of all field strength tensors appearing in MAG $\Longrightarrow$ rich structure.

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- Are there any interesting new effects that can emerge from this geometrical picture?