Theories with Torsion and Nonmetricity: From Cosmology to Astrophysics

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Phys. Rev. D **111** (2025) no.8, 084058;
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arXiv:2506.17017; arXiv:2507.02362;arXiv:2508.20035.

Outline

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1. Metric-Affine gravity and Gauge approach with Cubic interactions
2. Black holes with torsion and nonmetricity
   - Spherically symmetric black holes
   - Axially symmetric black holes
3. Algebraic Classification in MAG
4. Exact non-linear Gravitational waves with Torsion and Nonmetricity
5. Cosmology with torsion and nonmetricity
6. Main results and Possible directions

How to modify it?

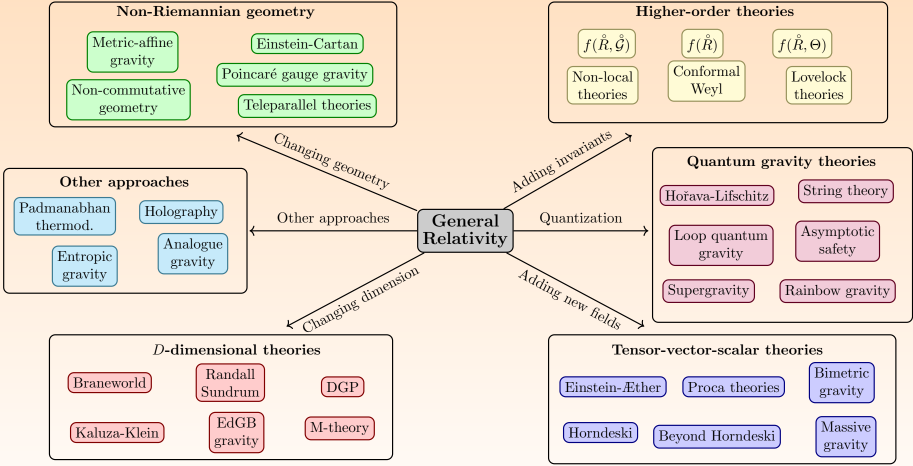

Figure: Classification of theories of gravity. (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

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  - New geometric degrees of freedom arise:

$$T^{\lambda}{}_{\mu\nu} = 2\tilde{\Gamma}^{\lambda}{}_{[\mu\nu]} \quad 24 \text{ dof} \quad \text{(Measures the nonclosure of infinitesimal parallelograms)}$$

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  - The general curvature is defined as in GR but changing $\Gamma^{\lambda}{}_{\mu\nu}$ to $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = \partial_{\mu}\tilde{\Gamma}^{\lambda}{}_{\rho\nu} - \partial_{\nu}\tilde{\Gamma}^{\lambda}{}_{\rho\mu} + \tilde{\Gamma}^{\lambda}{}_{\sigma\mu}\tilde{\Gamma}^{\sigma}{}_{\rho\nu} - \tilde{\Gamma}^{\lambda}{}_{\sigma\nu}\tilde{\Gamma}^{\sigma}{}_{\rho\mu}$$

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

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- Curvatures and field strengths:

$$\begin{aligned}
G_{ab\mu} &= \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac}g_{bd}e^{c\lambda}e^{d\rho}Q_{\mu\lambda\rho}\,,\\
F^a{}_{\mu\nu} &= \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\,e^b{}_\nu - \omega^a{}_{b\nu}\,e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,\\
F^a{}_{b\mu\nu} &= \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\,\omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\,\omega^c{}_{b\mu} = g_{bc}\,e^a{}_\lambda e^{c\rho}\tilde{R}^\lambda{}_{\rho\mu\nu}\,.
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- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity (PG).

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x \sqrt{-g} \Big[\mathcal{L}_m - \frac{1}{16\pi} \mathcal{L}_g \big(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q} \big) \Big] \, .$$

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- Correspondence between geometry and matter:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu\,,\\ \frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta \omega^a{}_{b\nu}} &= 16\pi\Delta_a{}^{b\nu}\,.\end{aligned}$$

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- Hypermomentum can be split into three parts:

$$\Delta_{\mu\nu\lambda} = {}^{(\mathrm{s})}\Delta_{[\mu\nu]\lambda} + \frac{1}{4}g_{\mu\nu}{}^{(\mathrm{d})}\Delta_\lambda + {}^{(\mathrm{sh})}\not{\Delta}_{(\mu\nu)\lambda}$$

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Quadratic Poincaré gauge theory - ghost issue

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+\, d_1\tilde{R}_{\mu\nu}\left(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\right) + \frac{1}{2}\left(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\right)\Big]\sqrt{-g}\,d^4x\,.$$

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$$+ d_1\tilde{R}_{\mu\nu}\left(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\right) + \frac{1}{2}\left(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\right)\Big]\sqrt{-g}\,d^4x\,.$$

- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_{μ} and S_{μ} propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\mathrm{curv-tors}} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^{\mu} T^{\nu} + h_2 \tilde{R} T_{\mu} T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^{\mu} S^{\nu} + h_4 \tilde{R} S_{\mu} S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_{\sigma} S^{\sigma} + h_{14} \varepsilon_{\nu}{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^{\mu} S^{\nu} + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_{\mu} S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t^{\alpha\tau}{}_{\gamma} + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau}\\
&+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^{\rho}{}_{\mu\tau\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\gamma\rho\tau}\\
&+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^{\mu}{}_{\gamma} S^{\alpha} t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_{\sigma} t^{\sigma\mu\nu}\,.
\end{aligned}$$

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\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu},\\
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- We showed that by including these Poincaré gauge invariants, ghost issue in the vector sector is solved!

Cubic Metric-Affine gauge theory

- Now let us include nonmetricity and decompose the fields as:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,,$$

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Cubic Metric-Affine gravity Lagrangian

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- We showed that by introducing these terms that are part of the Gauge approach, one can eliminate all known vector/axial ghosts.

Outline

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

$$\#24 \to \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

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- It is important to mention that the in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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Gauss-Bonnet with Dilations

- Now, consider a n-dimensional theory in a geometry without torsion and with $Q_{\lambda\mu\nu} = g_{\mu\nu}W_\lambda$:
  (S. Bahamonde and M. Bañados, Phys. Lett. B **869** (2025), 139869)

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\tilde{\mathcal{G}} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + (d-4)(d-3)G_{\alpha\beta}W^\alpha W^\beta \\
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- Using a 4-D regularised procedure, a 4-D black hole with primary hair was found (C. Charmousis, P. G. S. Fernandes and M. Hassaine, Phys. Rev. D **111** (2025) no.12, 12)

Electrodynamics coupled with torsion

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Electrodynamics coupled with torsion

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Theory with couplings between $F_{\mu\nu}$ and Torsion

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$$\Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(k_1 q^2 - \frac{1}{2}k_3\kappa_{\rm s} q - \frac{1}{32 k_2}k_3^2 q^2\right)\,, \quad A_\mu = \left(\frac{q}{r}, 0, 0, 0\right)\,.$$

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- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

3D Axially symmetric black holes - warm up exercise

- In 3D, Torsion has 9 dof: In the electromagnetism with torsion theory we found a slowly BTZ black hole solution:

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\begin{aligned}
ds^2 = &\Big(f_A(r) + J f_B(r)\Big)dt^2 - \Big(\frac{1}{f_A(r)} + J\,\frac{f_B(r)}{f_A^2}\Big)dr^2 - r^2 d\phi^2 \\
&+ 2\left(K_3\, J\, r^2 + \, J\,\frac{q_m f_A(r)}{q_e}\right)dt d\phi\,, \quad J \ll 1
\end{aligned}
$$

where

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f_A(r) &= -M - \Lambda r^2 - \left(2k_1 q_e^2 - \frac{k_3^2 q_e^2}{16 k_2} + \kappa_{\rm s} k_3 q_e\right)\log r\,, \\
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- We notice a new coupling between the intrinsic spin $\kappa_{\rm s}$, the angular momentum of the rotation J and the magnetic charge q_m, i.e. $\propto J q_m \kappa_{\rm s}$.

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

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- We initially focused on the search of black hole solutions that at least can display such an interaction in the gravitational action, regardless if it does not provide any modification in the metric tensor.
- We focused on a degenerate model of cubic PG theory, which provides static and spherically symmetric solutions with a spin charge that does not affect the Schwarzschild geometry

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Lagrangian like a gravitational spin-orbit interaction in the solution

$$\mathcal{L} = \frac{d_1 N_1^2 \kappa_{\rm s}^2}{8\pi r^4} + \mathcal{L}_{\rm SOI} = \frac{d_1 N_1^2 \kappa_{\rm s}^2}{8\pi r^4} + \frac{d_1 N_1 a \kappa_{\rm s} G(r,\vartheta)}{2\pi}\,.$$

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- The form of torsion contains 24 dof being non-zero.
- Is it possible to find a solution in the non-degenerate theory and find a modified Kerr metric with interactions between κ_s and a in the metric?

Axially symmetric black holes - spin orbit interaction

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- Is there any interesting new effect that can emerge from this solution and analogy with atomic physics?
- The arbitrary function $G(r,\vartheta)$ is encoded solely in the axial mode of the torsion tensor of the solution.
- Therefore, trajectories of Dirac particles minimally coupled to torsion will accordingly experience deviations from the geodesic motion, which can already be measured in the semiclassical limit by an acceleration of the form:

$$u^{\lambda}\nabla_{\lambda}u_{\mu}=\frac{1}{4m_{\rm s}}\hat{R}_{\lambda\rho\mu\nu}\bar{b}_0\sigma^{\lambda\rho}b_0u^{\nu}\,,$$

where u_μ represents the four-velocity of the particle, b_0 its normalised state, $m_{\rm s}$ its mass, $\sigma^{\lambda\rho}$ the spin matrices and $\hat{R}_{\lambda\rho\mu\nu}$ the part of the Riemann-Cartan curvature tensor that includes corrections from the axial mode alone.

Outline

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Riemmanian Curvature decomposition

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- **Result in Riemannian geometry:** Weyl has 6 types **(Petrov classification)**; Ricci traceless has 15 types**(Segre classification)**;
- **What happens in GR in vacuum?** $\not{R}_{\rho\nu} = R = 0$ and then the curvature is fully characterised by the Weyl tensor with their 6 types.

- This classification can be derived by means of its principal null directions which requires expressing any tensor in terms of a set of null vectors l_μ, k_μ, m_μ, and $\bar{m}_\mu$;

$$\begin{aligned} k^\mu l_\mu &= -m^\mu \bar{m}_\mu = 1\,, \\ k^\mu m_\mu &= k^\mu \bar{m}_\mu = l^\mu m_\mu = l^\mu \bar{m}_\mu = 0\,, \\ k^\mu k_\mu &= l^\mu l_\mu = m^\mu m_\mu = \bar{m}^\mu \bar{m}_\mu = 0\,. \end{aligned}$$

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Algebraic type	Segre characteristic	Intrinsic characterisation
I	$[1\,1\,1]$	$l_{[\sigma}\,{}^{(1)}\tilde{W}_{\lambda]\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
II	$[2\,1]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
D	$[(1\,1)\,1]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}k_{\omega]}k^\rho k^\mu = {}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
III	$[3]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\mu = 0$
N	$[(2\,1)]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu}l^\mu = 0$
O	$[-]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu} = 0$

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- The most general Type D solution in Einstein-Maxwell is known as the Plebański-Demiański characterised by $\{M, a, \alpha, N\}$ (mass, angular momentum, acceleration and Nut charge) and the electromagnetic charges.

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- The most general Type D solution in Einstein-Maxwell is known as the Plebański-Demiański characterised by $\{M, a, \alpha, N\}$ (mass, angular momentum, acceleration and Nut charge) and the electromagnetic charges.
- *Goldberg-Sachs theorem:* A vacuum solution of the Einstein's field equations admits a shear-free null geodesic congruence if and only if the conformal part of the Riemann tensor is algebraically special.

Curvature decomposition in Metric-Affine geometry

- First, we need to find the building blocks of the general curvature tensor (recall that in Riemannian geometry, we had 3: $W_{\alpha\beta\mu\nu}$, $\tilde{\not{R}}_{\mu\nu}$, and R).

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$$\tilde{R}_{\lambda\rho\mu\nu} = \tilde{R}_{[\lambda\rho]\mu\nu} + \tilde{R}_{(\lambda\rho)\mu\nu} := \tilde{W}_{\lambda\rho\mu\nu} + \tilde{Z}_{\lambda\rho\mu\nu}\,.$$

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General curvature

$$\tilde{R}_{\lambda\rho\mu\nu} = \tilde{R}_{[\lambda\rho]\mu\nu} + \tilde{R}_{(\lambda\rho)\mu\nu} := \tilde{W}_{\lambda\rho\mu\nu} + \tilde{Z}_{\lambda\rho\mu\nu}\,.$$

- We find that there are 11 building blocks S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **108** (2023) no.4, 4:

Building block	Number of independent components	Limit in Riemannian geometry
${}^{(1)}\tilde{Z}_{\lambda\rho\mu\nu}$	30	zero
${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu}$	10	Weyl tensor $W_{\lambda\rho\mu\nu}$
$\tilde{\not{R}}^{(T)}_{\lambda[\rho\mu\nu]}$	9	zero
$\tilde{\not{R}}^{(Q)}_{\lambda[\rho\mu\nu]}$	9	zero
$\tilde{\not{R}}_{(\mu\nu)}$	9	Ricci traceless $\not{R}_{\mu\nu}$
$\hat{\not{R}}^{(Q)}_{(\mu\nu)}$	9	zero
$\tilde{R}^{(T)}_{[\mu\nu]}$	6	zero
$\hat{R}^{(Q)}_{[\mu\nu]}$	6	zero
$\tilde{R}^{\lambda}{}_{\lambda\mu\nu}$	6	zero
$\tilde{R}$	1	Ricci scalar R
$*\tilde{R}$	1	zero

Algebraic classification in MAG

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  3. It is common that in spherical symmetry, the field strength tensors are of Type D (two null directions aligned). For a black hole solution endowed with shears, we found that even in spherical symmetry, ${}^{(1)}\tilde{Z}^{\lambda}{}_{\rho\mu\nu}$ is no longer Type D.

Outline

General Properties of *pp*-waves in General Relativity

- The *plane-fronted waves with parallel rays* (pp-waves) are exact solutions of Einstein's field equations describing gravitational waves propagating at the speed of light.

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- The line element can then be written in terms of Brinkmann coordinates (u, v, x, y) as:

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where $k^{\mu} = (0, 1, 0, 0)$.

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- By solving the GR's eqs one gets

$$(\partial_{xx} + \partial_{yy})\,H(u,x,y) = 0\,.$$

Physical Interpretation of pp waves

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- The ultra-relativistic limit of compact objects (e.g., boosting a Schwarzschild black hole) produces pp-wave geometries such as the Aichelburg–Sexl metric, relevant for understanding high-energy gravitational interactions.
- pp-waves play an important role to test the nonlinear properties of Einstein's equations, including wave–wave interactions, focusing effects on geodesics, and the potential formation of caustics and singularities.

pp waves in MAG

- In order to address the corresponding extension to Cubic MAG, the invariance defined by the wave vector can be directly imposed on the torsion and nonmetricity tensors:

$$
\begin{aligned}
T^{\lambda}{}_{\mu\nu} &= T^{\lambda}{}_{\mu\nu}(u,x,y)\,, \quad 24 \text{ dof} \\
Q_{\lambda\mu\nu} &= Q_{\lambda\mu\nu}(u,x,y)\,, \quad 40 \text{ dof}\,.
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- It is reasonable to impose Type N of algebraic conditions on the corresponding field strengths of the torsion and nonmetricity tensors:

$$
\begin{aligned}
& k^{\mu}\tilde{R}^{(T)}_{[\mu\nu]} = 0\,, \quad k_{[\rho}\tilde{R}^{(T)}_{\mu\nu]} = 0\,, \quad k^{\mu}\hat{R}^{(Q)}_{[\mu\nu]} = 0\,, \quad k_{[\rho}\hat{R}^{(Q)}_{\mu\nu]} = 0\,, \\
& k^{\mu}\tilde{R}^{\lambda}{}_{\lambda\mu\nu} = 0\,, \quad k_{[\rho|}\tilde{R}^{\lambda}{}_{\lambda|\mu\nu]} = 0\,, \quad k_{[\sigma}\tilde{\not{R}}^{(T)}_{\lambda][\rho\mu\nu]} = 0\,, \\
& k_{[\sigma}\tilde{\not{R}}^{(Q)}_{\lambda][\rho\mu\nu]} = 0\,, \quad k_{[\sigma}{}^{(1)}\tilde{Z}_{\lambda]\rho\mu\nu} = 0\,.
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- Additionally, we impose orthogonality of the wave null vector with torsion and nonmetricity, which can be expressed in terms of their respective vector, axial and tensor modes as:

$$k^\mu T_\mu = k^\mu S_\mu = k^\mu W_\mu = k^\mu \Lambda_\mu = 0\,,$$
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- Finally, we consider that the mentioned wave null vector is recurrent, which can be expressed in terms of its total covariant derivative as:

$$\tilde{\nabla}_\mu k^\nu = \left(\alpha T_\mu + \beta W_\mu + \gamma \Lambda_\mu\right) k^\nu\,,$$

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- This implies that its transport –with the full connection $\tilde{\nabla}$– along *any* possible curve is always parallel to itself, whereas its null character is also preserved if the orthogonality conditions hold.

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- The quantity $\mathcal{A}_\circ$ is invariant under rotations of the transverse plane, leading to scalar effects and representing a helicity-0 polarisation mode under the mentioned limit, but in this case induced by the dynamical torsion and nonmetricity fields.

Outline

Brief discussion on cosmology

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- The majority of these studies predominantly focus on torsion.
- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0\,.$$

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2 \gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\,\bar{n}_{[\mu}\bar{P}_{\nu]}{}^\lambda + 2T_2(t)\,\bar{\varepsilon}^\lambda{}_{\mu\nu\rho}\bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t)\bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t)\bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t)\bar{P}_{\lambda(\mu}\bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

Cosmology in MAG - matter

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

$$T_{\mu\nu} = (\rho(t) + p(t))\, n_\mu n_\nu + p(t) g_{\mu\nu} = \rho(t) n_\mu n_\nu + p(t) p_{\mu\nu}\,,$$

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3}\Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4}\Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho\,,$$

which contains 5 different sources dof related to the intrinsic spin, dilations, and shears.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_\mu T^\mu{}_\alpha - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_\mu\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\alpha{}^{\mu\nu}) + 2T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) = 0\,.$$

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- When Nonmetricity is vanishing, only intrinsic spin contributes

$$\Delta_{[\lambda\mu]\nu} = 2\,{}^{(\mathrm{s})}\Delta_3 n_{[\lambda}P_{\mu]\nu} + {}^{(\mathrm{s})}\Delta_5\varepsilon_{\lambda\mu\nu\rho}n^\rho\,, \quad {}^{(\mathrm{d})}\Delta_4 = {}^{(\mathrm{sh})}\Delta_1 = {}^{(\mathrm{sh})}\Delta_2 = 0\,,$$

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- In our Cubic Poincaré Gauge Gravity theory, we have the modified FLRW equations of the form: S. Bahamonde, R. Briffa, K. Dialektopoulos, D. Iosifidis and J. Levi Said, arXiv:2506.17017.

$$3H^2 + f(T_1(t),T_2(t)) \;=\; \kappa^2\rho\,, \quad 3H^2 + 2\dot{H} + g(T_1(t),T_2(t)) = -\kappa^2 p\,.$$

and the connection equations:

$$\begin{aligned}
2\kappa^2\,{}^{(\mathrm{s})}\Delta_3 &= 3T_1\Big[6h_1(H-T_1)(H-2T_1) - 6(h_1-16h_{13})T_2^2 + m_T^2\Big]\,,\\
\kappa^2\,{}^{(\mathrm{s})}\Delta_5 &= 3T_2\Big[48h_{13}\left(-H^2+T_1^2+2T_2^2\right) - 3h_1T_1^2 + 8m_S^2\Big]\,.
\end{aligned}$$

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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T.

- The conservation equation is modified:

$$\dot{\rho} + 3H(\rho + p) = 3\,^{(\mathrm{s})}\Delta_5\left(HT_2 + \dot{T}_2\right) - 3\,^{(\mathrm{s})}\Delta_3\left(\dot{H} - HT_1 + H^2 - \dot{T}_1\right)\,.$$

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- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.
- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

SVT decomposition around FLRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(\mathrm{T})}, h_i^{(\mathrm{T})}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(\mathrm{TT})}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(\mathrm{T})}, B_i^{(\mathrm{T})}, A_i^{(\mathrm{T})}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(\mathrm{T})}, \mathcal{B}_i^{(\mathrm{T})}, \mathcal{A}_i^{(\mathrm{T})}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(\mathrm{TT})}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(\mathrm{TT})}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FLRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta, \psi, \xi, \Lambda, Y, Z, \kappa, Q, W, C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y}, \mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\mathrm{T})}{}_i, Y^{(\mathrm{T})}{}_i, Z^{(\mathrm{T})}{}_i, \kappa^{(\mathrm{T})}{}_i, Q^{(\mathrm{T})}{}_i, W^{(\mathrm{T})}{}_i, C^{(\mathrm{T})}{}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(\mathrm{T})}{}_i, \mathcal{Q}^{(\mathrm{T})}{}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(\mathrm{TT})}{}_{ij}, Q^{(\mathrm{TT})}{}_{ij}, C^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	6
1 rank-2 pseudotensor	$\{\mathcal{Q}^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	2
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- Since we have spin-3,spin-2,spin-1,spin-0 being dynamical, different effects might emerge!

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- Are there any interesting new effects that can emerge from this geometrical picture?