Theories with Torsion and Nonmetricity: From Cosmology to Astrophysics

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Mainly jointly with Jorge Gigante Valcarcel

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Phys. Rev. D **110** (2024) no.12, 12
Phys. Rev. D **111** (2025) no.8, 084058;
Phys. Lett. B **869** (2025), 139869;
arXiv:2506.17017; arXiv:2507.02362;arXiv:2508.20035.

Outline

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How to modify it?

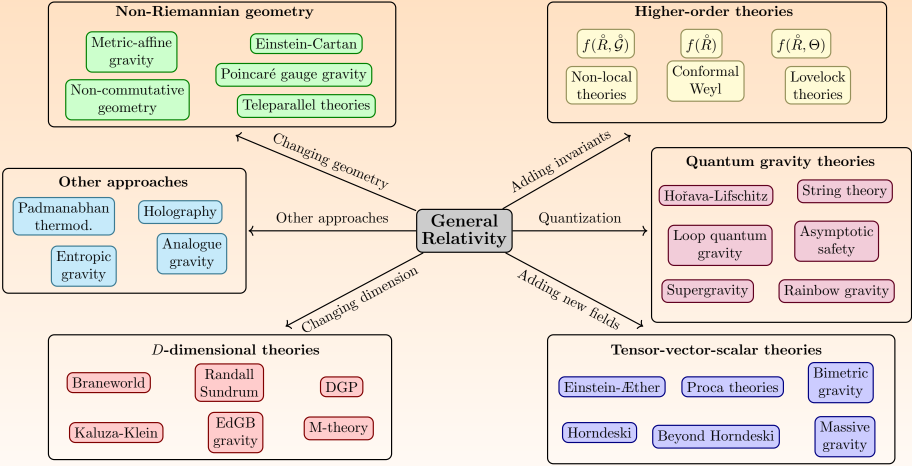

Figure: Classification of theories of gravity. (S. Bahamonde, K. F. Dialektopoulos, C. Escamilla-Rivera, G. Farrugia, V. Gakis, M. Hendry, M. Hohmann, J. Levi Said, J. Mifsud and E. Di Valentino, "Teleparallel gravity: from theory to cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.)

Definitions and conventions

- The metric tensor $g_{\mu\nu}$ constitutes a natural isomorphism between the tangent and cotangent spaces defined at any point of the differentiable manifold

$$U_\mu = g_{\mu\nu} U^\nu \,,$$

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- Thus it allows the measurement of infinitesimal distances

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \,,$$

as well as of vector lengths and angles among them

$$||\mathbf{U}|| = \sqrt{|g_{\mu\nu} U^\mu U^\nu|}, \quad \cos\alpha \qquad = \frac{g_{\mu\nu} m^\mu n^\nu}{\sqrt{|g_{\alpha\beta} m^\alpha m^\beta|}\sqrt{|g_{\gamma\sigma} n^\gamma n^\sigma|}} \,.$$

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- The affine/linear connection $\tilde{\Gamma}^{\rho}{}_{\lambda\mu}$ defines the covariant derivative operator

$$\tilde{\nabla}_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \tilde{\Gamma}^{\nu}{}_{\lambda\mu}V^{\lambda}\,,$$
$$\tilde{\nabla}_{\mu}V^{\nu} \xrightarrow{GCT} \tilde{\nabla}'_{\mu}V'^{\nu} = \Lambda_{\mu}{}^{\lambda}\Lambda^{\nu}{}_{\rho}\tilde{\nabla}_{\lambda}V^{\rho}\,,$$

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- Commutation rule:

$$[\tilde{\nabla}_{\mu},\tilde{\nabla}_{\nu}]\,v^{\lambda} = \tilde{R}^{\lambda}{}_{\rho\mu\nu}\,v^{\rho} + T^{\rho}{}_{\mu\nu}\,\tilde{\nabla}_{\rho}v^{\lambda}\,.$$

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  - The general curvature is defined as in GR but changing $\Gamma^{\lambda}{}_{\mu\nu}$ to $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$:

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

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- Recall that one can "go back" to the standard curvature by contracting this quantity with tetrads:

$$\tilde{R}^\lambda{}_{\rho\mu\nu} = e_a{}^\lambda e^b{}_\rho \, F^a{}_{b\mu\nu}$$

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- In particular, it measures the change of vector components on parallel transport along an infinitesimal closed curve:

$$\delta V_{\mu} = \tilde{R}^{\lambda}{}_{\mu\rho\nu} V_{\lambda} ds^{\rho\nu} \,,$$

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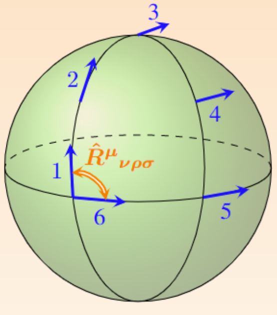

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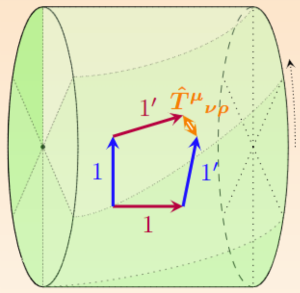

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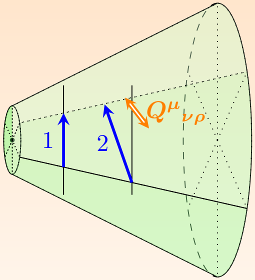

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\tilde{\nabla}_{[\sigma|}\tilde{R}^{\lambda}{}_{\rho|\mu\nu]} &= T^{\omega}{}_{[\sigma\mu|}\tilde{R}^{\lambda}{}_{\rho\omega|\nu]}\,,\\
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- Three independent rank-2 tensors defined from the first contractions of the curvature tensor:

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- Unique scalar and pseudoscalar curvatures:

$$\tilde{R} = g^{\mu\nu}\tilde{R}_{\mu\nu}\,, \quad *\tilde{R} = \varepsilon^{\lambda\rho\mu\nu}\tilde{R}_{\lambda\rho\mu\nu}\,.$$

Post-Riemannian decomposition

- It is useful to separate the connection as

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- Curvatures and field strengths:

$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac} g_{bd} e^{c\lambda} e^{d\rho} Q_{\mu\lambda\rho}\,,$$
$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\, e^b{}_\nu - \omega^a{}_{b\nu}\, e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,$$
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$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\,e^b{}_\nu - \omega^a{}_{b\nu}\,e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,$$
$$F^a{}_{b\mu\nu} = \partial_\mu\omega^a{}_{b\nu} - \partial_\nu\omega^a{}_{b\mu} + \omega^a{}_{c\mu}\,\omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\,\omega^c{}_{b\mu} = g_{bc}\,e^a{}_\lambda e^{c\rho}\tilde{R}^\lambda{}_{\rho\mu\nu}\,.$$

- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity (PG).

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x\sqrt{-g}\Big[\mathcal{L}_m - \frac{1}{16\pi}\mathcal{L}_g\big(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q}\big)\Big]\,.$$

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- Correspondence between geometry and matter:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu\,,\\ \frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta\omega^a{}_{b\nu}} &= 16\pi\Delta_a{}^{b\nu}\,.\end{aligned}$$

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Conservation equation - Only Torsion

- $\theta_a{}^{\nu}$ is the canonical energy-momentum tensor that can be related to the energy-momentum tensor as

$$\Theta^{\mu}{}_{\lambda} = T^{\mu}{}_{\lambda} + \frac{1}{2\sqrt{-g}}\left[\tilde{\nabla}_{\nu}(\sqrt{-g}\Delta_{\lambda}{}^{\mu\nu}) - 2\sqrt{-g}\,T_{\nu}\Delta_{\lambda}{}^{\mu\nu}\right].$$

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- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation:

$$\nabla_\mu T^\mu{}_\alpha - \frac{1}{2}\Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha} + \frac{1}{2}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\Delta_\alpha{}^{\mu\nu} + T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu\Delta_\lambda{}^{\mu\nu} = 0\,.$$

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- Obviously, if there is no hypermomentum, we find the standard energy-momentum conservation $\nabla_\mu T^\mu{}_\alpha = 0$.

Quadratic Poincaré gauge theory - ghost issue

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$S_g = \frac{1}{16\pi}\int\Big[-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu}$$
$$+\, d_1\tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2 T_{\mu}T^{\mu} + m_S^2 S_{\mu}S^{\mu} + m_t^2 t_{\lambda\mu\nu}t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\,d^4x\,.$$

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- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_{μ} and S_{μ} propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^{\mu} T^{\nu} + h_2 \tilde{R} T_{\mu} T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^{\mu} S^{\nu} + h_4 \tilde{R} S_{\mu} S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_{\sigma} S^{\sigma} + h_{14} \varepsilon_{\nu}{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^{\mu} S^{\nu} + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_{\mu} S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu},\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t^{\alpha\tau}{}_{\gamma} + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau}\\
&+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^{\rho}{}_{\mu\tau\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\gamma\rho\tau}\\
&+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^{\mu}{}_{\gamma} S^{\alpha} t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_{\sigma} t^{\sigma\mu\nu}\,.
\end{aligned}$$

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\end{aligned}$$

- We showed that by including these Poincaré gauge invariants, ghost issue in the vector sector is solved!

Cubic Metric-Affine gauge theory

- Now let us include nonmetricity and decompose the fields as:

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,,$$

$$Q_{\lambda\mu\nu} = g_{\mu\nu}W_{\lambda} + \frac{1}{2}\left(g_{\lambda\mu}\Lambda_{\nu} + g_{\lambda\nu}\Lambda_{\mu}\right) - \frac{1}{4}g_{\mu\nu}\Lambda_{\lambda} + \frac{1}{3}\varepsilon_{\lambda\rho\sigma(\mu}\Omega_{\nu)}{}^{\rho\sigma} + q_{\lambda\mu\nu}\,.$$

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Cubic Metric-Affine gravity Lagrangian

$$16\pi\mathcal{L} = \mathcal{L}_{\rm Quad} + \mathcal{L}^{(3)}_{\rm RTT} + \mathcal{L}^{(3)}_{\rm RQQ} + \mathcal{L}^{(3)}_{\rm RTQ}\,.$$

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  - From torsion: 1 vector T_{μ} and 1 axial vector S_{μ}
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- Our Cubic MAG theory can be expressed as (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) no.8, 084058)

Cubic Metric-Affine gravity Lagrangian

$$16\pi\mathcal{L} = \mathcal{L}_{\rm Quad} + \mathcal{L}^{(3)}_{\rm RTT} + \mathcal{L}^{(3)}_{\rm RQQ} + \mathcal{L}^{(3)}_{\rm RTQ}\,.$$

- We showed that by introducing these terms that are part of the Gauge approach, one can eliminate all known vector/axial ghosts.

Outline

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

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- It is important to mention that the in this theory both torsion and nonmetricity are propagating and moreover, their spin-2 and spin-3 parts are dynamical.
- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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- Now, consider a n-dimensional theory in a geometry without torsion and with $Q_{\lambda\mu\nu} = g_{\mu\nu}W_\lambda$:
  (S. Bahamonde and M. Bañados, Phys. Lett. B **869** (2025), 139869)

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- Using a 4-D regularised procedure, a 4-D black hole with primary hair was found (C. Charmousis, P. G. S. Fernandes and M. Hassaine, Phys. Rev. D **111** (2025) no.12, 12)

Electrodynamics coupled with torsion

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- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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$$\mathcal{L}_{SO} = \lambda(r)\,\mathbf{L}\cdot\mathbf{S} \propto \alpha\lambda(r)\cos\vartheta$$

  where $\mathbf{L}$ is the orbital angular momentum, $\mathbf{S}$ is the spin angular momentum, and $\lambda(r)$ is a coupling function.

- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

Axially symmetric black holes - spin orbit interaction

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- We focused on a degenerate model of cubic PG theory, which provides static and spherically symmetric solutions with a spin charge that does not affect the Schwarzschild geometry

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Lagrangian like a gravitational spin-orbit interaction in the solution

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- The form of torsion contains 24 dof being non-zero.
- Is it possible to find a solution in the non-degenerate theory and find a modified Kerr metric with interactions between κ_s and a in the metric?

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- The arbitrary function $G(r,\vartheta)$ is encoded solely in the axial mode of the torsion tensor of the solution.
- Therefore, trajectories of Dirac particles minimally coupled to torsion will accordingly experience deviations from the geodesic motion, which can already be measured in the semiclassical limit by an acceleration of the form:

$$u^\lambda \nabla_\lambda u_\mu = \frac{1}{4m_{\rm s}} \hat{R}_{\lambda\rho\mu\nu} \bar{b}_0 \sigma^{\lambda\rho} b_0 u^\nu \,,$$

where u_μ represents the four-velocity of the particle, b_0 its normalised state, $m_{\rm s}$ its mass, $\sigma^{\lambda\rho}$ the spin matrices and $\hat{R}_{\lambda\rho\mu\nu}$ the part of the Riemann-Cartan curvature tensor that includes corrections from the axial mode alone.

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- **Result in Riemannian geometry:** Weyl has 6 types **(Petrov classification)**; Ricci traceless has 15 types**(Segre classification)**;
- **What happens in GR in vacuum?** $\not{R}_{\rho\nu} = R = 0$ and then the curvature is fully characterised by the Weyl tensor with their 6 types.

- This classification can be derived by means of its principal null directions which requires expressing any tensor in terms of a set of null vectors l_μ, k_μ, m_μ, and $\bar{m}_\mu$;

$$\begin{aligned} k^\mu l_\mu &= -m^\mu \bar{m}_\mu = 1\,, \\ k^\mu m_\mu &= k^\mu \bar{m}_\mu = l^\mu m_\mu = l^\mu \bar{m}_\mu = 0\,, \\ k^\mu k_\mu &= l^\mu l_\mu = m^\mu m_\mu = \bar{m}^\mu \bar{m}_\mu = 0\,. \end{aligned}$$

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Algebraic type	Segre characteristic	Intrinsic characterisation
I	$[1\,1\,1]$	$l_{[\sigma}\,{}^{(1)}\tilde{W}_{\lambda]\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
II	$[2\,1]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
D	$[(1\,1)\,1]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}k_{\omega]}k^\rho k^\mu = {}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\rho l^\mu = 0$
III	$[3]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu[\nu}l_{\omega]}l^\mu = 0$
N	$[(2\,1)]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu}l^\mu = 0$
O	$[-]$	${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu} = 0$

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- The most general Type D solution in Einstein-Maxwell is known as the Plebański-Demiański characterised by $\{M, a, \alpha, N\}$ (mass, angular momentum, acceleration and Nut charge) and the electromagnetic charges.
- *Goldberg-Sachs theorem:* A vacuum solution of the Einstein's field equations admits a shear-free null geodesic congruence if and only if the conformal part of the Riemann tensor is algebraically special.

Curvature decomposition in Metric-Affine geometry

- First, we need to find the building blocks of the general curvature tensor (recall that in Riemannian geometry, we had 3: $W_{\alpha\beta\mu\nu}$, $\tilde{\not{R}}_{\mu\nu}$, and R).

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$$\tilde{R}_{\lambda\rho\mu\nu} = \tilde{R}_{[\lambda\rho]\mu\nu} + \tilde{R}_{(\lambda\rho)\mu\nu} := \tilde{W}_{\lambda\rho\mu\nu} + \tilde{Z}_{\lambda\rho\mu\nu}\,.$$

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- We find that there are 11 building blocks S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **108** (2023) no.4, 4:

Building block	Number of independent components	Limit in Riemannian geometry
${}^{(1)}\tilde{Z}_{\lambda\rho\mu\nu}$	30	zero
${}^{(1)}\tilde{W}_{\lambda\rho\mu\nu}$	10	Weyl tensor $W_{\lambda\rho\mu\nu}$
$\tilde{\not{R}}^{(T)}_{\lambda[\rho\mu\nu]}$	9	zero
$\tilde{\not{R}}^{(Q)}_{\lambda[\rho\mu\nu]}$	9	zero
$\tilde{\not{R}}_{(\mu\nu)}$	9	Ricci traceless $\not{R}_{\mu\nu}$
$\hat{\not{R}}^{(Q)}_{(\mu\nu)}$	9	zero
$\tilde{R}^{(T)}_{[\mu\nu]}$	6	zero
$\hat{R}^{(Q)}_{[\mu\nu]}$	6	zero
$\tilde{R}^{\lambda}{}_{\lambda\mu\nu}$	6	zero
$\tilde{R}$	1	Ricci scalar R
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  3. It is common that in spherical symmetry, the field strength tensors are of Type D (two null directions aligned). For a black hole solution endowed with shears, we found that even in spherical symmetry, ${}^{(1)}\tilde{Z}^{\lambda}{}_{\rho\mu\nu}$ is no longer Type D.

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- Usually, it is possible to avoid cosmological singularities and replace them a cosmic bounce.
- Dark energy can be explained by the scalar modes of torsion.
- It is possible to find inflationary models such as Einstein-Cartan couple to Higgs or other more complicated ones which are compatible with observations.

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

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- By solving those equations one get the FLRW metric and torsion+nonmetricity satisfying homogeneity and isotropy:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu}\mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2 \gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\,\bar{n}_{[\mu}\bar{P}_{\nu]}{}^\lambda + 2T_2(t)\,\bar{\varepsilon}^{\,\lambda}{}_{\mu\nu\rho}\bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t)\bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t)\bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t)\bar{P}_{\lambda(\mu}\bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij}\mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2 \mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

Cosmology in MAG - matter

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3}\Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4}\Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho\,,$$

which contains 5 different sources dof related to the intrinsic spin, dilations, and shears.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_\mu T^\mu{}_\alpha - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_\mu\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\alpha{}^{\mu\nu}) + 2T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) = 0\,.$$

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$$3H^2 + f(T_1(t),T_2(t)) \;=\; \kappa^2\rho\,,\quad 3H^2 + 2\dot{H} + g(T_1(t),T_2(t)) = -\kappa^2 p\,.$$

and the connection equations:

$$\begin{aligned} 2\kappa^2\,{}^{(\mathrm{s})}\Delta_3 &= 3T_1\left[6h_1(H-T_1)(H-2T_1) - 6(h_1-16h_{13})T_2^2 + m_T^2\right], \\ \kappa^2\,{}^{(\mathrm{s})}\Delta_5 &= 3T_2\left[48h_{13}\left(-H^2+T_1^2+2T_2^2\right) - 3h_1T_1^2 + 8m_S^2\right]. \end{aligned}$$

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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T.

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- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.
- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

SVT decomposition around FLRW

- The 10 dof described by the metric perturbations are split in terms of four scalars $\{\alpha, \beta, \zeta, h\}$ (1 dof each), two transverse vectors $\{\beta_i^{(\mathrm{T})}, h_i^{(\mathrm{T})}\}$ (2 dof each), and one symmetric and transverse-traceless tensor $h_{ij}^{(\mathrm{TT})}$ (2 dof).

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SVT	Quantities	dof	Total dof
5 scalars	$\{T, B, \phi, A, \varrho\}$	1 dof each	5
3 pseudoscalars	$\{\mathcal{S}, \mathcal{B}, \mathcal{A}\}$	1 dof each	3
3 vectors	$\{T_i^{(\mathrm{T})}, B_i^{(\mathrm{T})}, A_i^{(\mathrm{T})}\}$	2 dof each	6
3 pseudovectors	$\{\mathcal{S}_i^{(\mathrm{T})}, \mathcal{B}_i^{(\mathrm{T})}, \mathcal{A}_i^{(\mathrm{T})}\}$	2 dof each	6
1 rank-2 tensor	$\{A_{ij}^{(\mathrm{TT})}\}$	2 dof each	2
1 rank-2 pseudotensor	$\{\mathcal{A}_{ij}^{(\mathrm{TT})}\}$	2 dof each	2

Table: Perturbation spectrum for the torsion tensor. K. Aoki, S. Bahamonde, J. Gigante Valcarcel and M. A. Gorji, "Cosmological Perturbation Theory in Metric-Affine Gravity," Phys. Rev. D **110** (2024) no.2, 2.

SVT decomposition around FLRW

SVT	Quantities	dof	Total dof
10 scalars	$\{\theta,\psi,\xi,\Lambda,Y,Z,\kappa,Q,W,C\}$	1 dof each	10
2 pseudoscalars	$\{\mathcal{Y},\mathcal{Q}\}$	1 dof each	2
7 vectors	$\{\Lambda^{(\mathrm{T})}{}_i, Y^{(\mathrm{T})}{}_i, Z^{(\mathrm{T})}{}_i, \kappa^{(\mathrm{T})}{}_i, Q^{(\mathrm{T})}{}_i, W^{(\mathrm{T})}{}_i, C^{(\mathrm{T})}{}_i\}$	2 dof each	14
2 pseudovectors	$\{\mathcal{Y}^{(\mathrm{T})}{}_i, \mathcal{Q}^{(\mathrm{T})}{}_i\}$	2 dof each	4
3 rank-2 tensor	$\{\kappa^{(\mathrm{TT})}{}_{ij}, Q^{(\mathrm{TT})}{}_{ij}, C^{(\mathrm{TT})}{}_{ij}\}$	2 dof each	6
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- Since we have spin-3,spin-2,spin-1,spin-0 being dynamical, different effects might emerge!

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- Developed **cosmological perturbation theory** including helicity-3 sector of spin-3 nonmetricity.
- We formulated the algebraic classification of all field strength tensors appearing in MAG $\Longrightarrow$ rich structure.

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- Are there any interesting new effects that can emerge from this geometrical picture?