Metric-Affine Theories of Gravity: From theory to new possible effects

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Outline

1. Metric-Affine gravity and Gauge approach with Cubic interactions
2. Black holes with torsion and nonmetricity
   - Spherically symmetric black holes
   - Axially symmetric black holes
3. Main results

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  - New geometric degrees of freedom arise:

$$T^{\lambda}{}_{\mu\nu} = 2\tilde{\Gamma}^{\lambda}{}_{[\mu\nu]} \quad 24 \text{ dof}$$ (Measures the nonclosure of infinitesimal parallelograms)

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  - The general curvature is defined as in GR but changing $\Gamma^{\lambda}{}_{\mu\nu}$ to $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = \partial_{\mu}\tilde{\Gamma}^{\lambda}{}_{\rho\nu} - \partial_{\nu}\tilde{\Gamma}^{\lambda}{}_{\rho\mu} + \tilde{\Gamma}^{\lambda}{}_{\sigma\mu}\tilde{\Gamma}^{\sigma}{}_{\rho\nu} - \tilde{\Gamma}^{\lambda}{}_{\sigma\nu}\tilde{\Gamma}^{\sigma}{}_{\rho\mu}$$

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

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- Curvatures and field strengths:

$$G_{ab\mu} = \partial_\mu g_{ab} - g_{ac}\,\omega^c{}_{b\mu} - g_{bc}\,\omega^c{}_{a\mu} = g_{ac} g_{bd} e^{c\lambda} e^{d\rho} Q_{\mu\lambda\rho}\,,$$
$$F^a{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu}\, e^b{}_\nu - \omega^a{}_{b\nu}\, e^b{}_\mu = e^a{}_\lambda T^\lambda{}_{\nu\mu}\,,$$
$$F^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{b\nu} - \partial_\nu \omega^a{}_{b\mu} + \omega^a{}_{c\mu}\, \omega^c{}_{b\,\nu} - \omega^a{}_{c\nu}\, \omega^c{}_{b\mu} = g_{bc}\, e^a{}_\lambda e^{c\rho} \tilde{R}^\lambda{}_{\rho\mu\nu}\,.$$

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- When nonmetricity is vanishing, the group becomes the Poincaré group$\Longrightarrow$ Poincaré gauge theories of gravity.

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

$$S = \int d^4x\sqrt{-g}\Big[\mathcal{L}_m - \frac{1}{16\pi}\mathcal{L}_g\big(\tilde{\mathcal{R}}, \mathcal{T}, \mathcal{Q}\big)\Big]\,.$$

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- Correspondence between geometry and matter:

$$\begin{aligned}\frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta e^a{}_\nu} &= 16\pi\theta_a{}^\nu\,,\\ \frac{1}{\sqrt{-g}}\frac{\delta\left(\mathcal{L}_g\sqrt{-g}\right)}{\delta \omega^a{}_{b\nu}} &= 16\pi\Delta_a{}^{b\nu}\,.\end{aligned}$$

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- Hypermomentum can be split into three parts:

$$\Delta_{\mu\nu\lambda} = {}^{(\mathrm{s})}\Delta_{[\mu\nu]\lambda} + \frac{1}{4}g_{\mu\nu}{}^{(\mathrm{d})}\Delta_\lambda + {}^{(\mathrm{sh})}\,\check{\Delta}_{(\mu\nu)\lambda}$$

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Quadratic Poincaré gauge theory - ghost issue

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu} T_{\mu} - \delta^{\lambda}{}_{\mu} T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu} S^{\rho} + t^{\lambda}{}_{\mu\nu} \,.$$

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- The most general class of quadratic Poincaré gauge models that are reduced to General Relativity in the absence of torsion is:

$$\begin{aligned} S_g = \frac{1}{16\pi}\int\Big[&-R + c_2\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\mu\rho\nu} - \frac{1}{2}\left(2c_1 + c_2\right)\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\mu\nu\lambda\rho} + c_1\tilde{R}_{\lambda\rho\mu\nu}\tilde{R}^{\lambda\rho\mu\nu} \\ &+ d_1\tilde{R}_{\mu\nu}\big(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\big) + \frac{1}{2}\big(m_T^2T_{\mu}T^{\mu} + m_S^2S_{\mu}S^{\mu} + m_t^2t_{\lambda\mu\nu}t^{\lambda\mu\nu}\big)\Big]\sqrt{-g}\,d^4x\,. \end{aligned}$$

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$$+ d_1 \tilde{R}_{\mu\nu}\left(\tilde{R}^{\mu\nu} - \tilde{R}^{\nu\mu}\right) + \frac{1}{2}\left(m_T^2 T_{\mu} T^{\mu} + m_S^2 S_{\mu} S^{\mu} + m_t^2 t_{\lambda\mu\nu} t^{\lambda\mu\nu}\right)\Big]\sqrt{-g}\, d^4x\,.$$

- It is not possible to have a stable propagating torsion tensor in quadratic Poincaré gauge theory for general backgrounds. Kinetic part of vectors T_{μ} and S_{μ} propagate a ghost.

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^{\mu} T^{\nu} + h_2 \tilde{R} T_{\mu} T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^{\mu} S^{\nu} + h_4 \tilde{R} S_{\mu} S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_{\sigma} S^{\sigma} + h_{14} \varepsilon_{\nu}{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^{\mu} S^{\nu} + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_{\mu} S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}St} &= h_{20} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t^{\alpha\tau}{}_{\gamma} + h_{21} \varepsilon_{\alpha\rho\mu\nu} \tilde{R}_{\tau}{}^{\rho\mu\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau}\\
&+ h_{22} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}^{\rho}{}_{\mu\tau\nu} S^{\gamma} t_{\gamma}{}^{\alpha\tau} + h_{23} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\gamma\rho\tau}\\
&+ h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^{\mu}{}_{\gamma} S^{\alpha} t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_{\sigma} t^{\sigma\mu\nu}\,.
\end{aligned}$$

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&\quad + h_{24} \varepsilon_{\alpha\rho}{}^{\mu\nu} \tilde{R}_{\gamma\mu\tau\nu} S^{\alpha} t^{\rho\tau\gamma} + h_{25} \varepsilon_{\alpha\rho\tau\mu} \tilde{R}^{\mu}{}_{\gamma} S^{\alpha} t^{\rho\tau\gamma} + h_{26} \varepsilon_{\lambda\rho\mu\nu} \tilde{R}^{\lambda\rho} S_{\sigma} t^{\sigma\mu\nu}\,.
\end{aligned}$$

- We showed that by including these Poincaré gauge invariants, ghost issue is solved!

Cubic Poincaré gauge theory

- Cubic parity preserving branch with mixing terms: (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

$$\mathcal{L}^{(3)}_{\rm curv-tors} = \mathcal{L}^{(3)}_{\tilde{R}TT} + \mathcal{L}^{(3)}_{\tilde{R}SS} + \mathcal{L}^{(3)}_{\tilde{R}tt} + \mathcal{L}^{(3)}_{\tilde{R}TS} + \mathcal{L}^{(3)}_{\tilde{R}Tt} + \mathcal{L}^{(3)}_{\tilde{R}St}\,,$$

$$\begin{aligned}
\mathcal{L}^{(3)}_{\tilde{R}TT} &= h_1 \tilde{R}_{\mu\nu} T^{\mu} T^{\nu} + h_2 \tilde{R} T_{\mu} T^{\mu}\,, \quad \mathcal{L}^{(3)}_{\tilde{R}SS} = h_3 \tilde{R}_{\mu\nu} S^{\mu} S^{\nu} + h_4 \tilde{R} S_{\mu} S^{\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}tt} &= h_5 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\rho} t^{\sigma\mu\nu} + h_6 \tilde{R}_{\lambda\rho\mu\nu} t_{\sigma}{}^{\lambda\mu} t^{\sigma\rho\nu} + h_7 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\rho}{}_{\sigma} t^{\sigma\mu\nu}\\
&+ h_8 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\sigma\rho\nu} + h_9 \tilde{R}_{\lambda\rho\mu\nu} t^{\lambda\mu}{}_{\sigma} t^{\rho\nu\sigma} + h_{10} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\rho\mu\nu}\\
&+ h_{11} \tilde{R}_{\lambda\rho} t_{\mu\nu}{}^{\lambda} t^{\mu\nu\rho} + h_{12} \tilde{R} t_{\lambda\rho\mu} t^{\lambda\rho\mu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}TS} &= h_{13} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho\mu\nu} T_{\sigma} S^{\sigma} + h_{14} \varepsilon_{\nu}{}^{\lambda\rho\sigma} \tilde{R}_{\lambda\rho\mu\sigma} T^{\mu} S^{\nu} + h_{15} \varepsilon^{\lambda\rho\mu\nu} \tilde{R}_{\lambda\rho} T_{\mu} S_{\nu}\,,\\
\mathcal{L}^{(3)}_{\tilde{R}Tt} &= h_{16} \tilde{R}_{\lambda\rho\mu\nu} T^{\nu} t^{\lambda\rho\mu} + h_{17} \tilde{R}_{\lambda\rho\mu\nu} T^{\rho} t^{\lambda\mu\nu} + h_{18} \tilde{R}_{\lambda\rho} T_{\mu} t^{\mu\lambda\rho} + h_{19} \tilde{R}_{\lambda\rho} T_{\mu} t^{\lambda\rho\mu},\\
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\end{aligned}$$

- We showed that by including these Poincaré gauge invariants, ghost issue is solved!
- Further, we showed that nonmetricity can be added dynamically as a ghost-free Gauge theory(S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) no.8, 084058)

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^\lambda{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^\lambda{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\, dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2 \left(d\vartheta^2 + \sin^2\vartheta d\varphi^2 \right) \right. \;;$$

$$\#24 \to \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \to \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

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New exact black hole solution with three intrinsic charges

$$g_{tt} = -\frac{1}{g_{rr}} = \Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(H_1\kappa_{\rm s}^2 + H_2\kappa_{\rm d}^2 + H_3\kappa_{\rm sh}^2\right).$$

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- All the masses of the tensor modes of torsion and nonmetricity are different from zero $\Longrightarrow$ We evaded the Weinberg-Witten no-go theorem (massless higher-spin fields are pathological)

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5. The intrinsic shear charge $\kappa_{\rm d}$: deformations that change the shape of an object without changing its volume.

Axially symmetric black holes - spin orbit interaction

Spin-Orbit Interaction in Atomic Physics

The spin-orbit interaction is a fundamental quantum mechanical effect that describes how an electron's intrinsic spin interacts with its orbital motion around the nucleus. This interaction leads to energy level splitting in atoms, particularly noticeable in heavy elements.

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- The spin-orbit interaction increases the energy gap between certain nuclear energy levels, making nuclei with magic numbers more stable.

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- The form of torsion contains 24 dof being non-zero.
- Is there any interesting new effect that can emerge from this analogy?

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  - Cubic MAG: dynamical T, Q, massive spin-2 and spin-3 with intrinsic charges: **spin, dilation, shear**.
  - Cubic Poincaré: slowly rotating Kerr-like BH with **gravitational spin-orbit interaction**: $\mathcal{L} \propto \frac{1}{16\pi G r^6} a d_1 \kappa_s m(\alpha \cos\vartheta + \beta \sin\vartheta)$.