Geometric Hair and Gravitational Waves in Theories with Torsion and Nonmetricity

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Senior Research Fellow at Institute for Basic Science, Daejeon, South Korea.
KIAA PKU seminar, 18/May/2026

Outline

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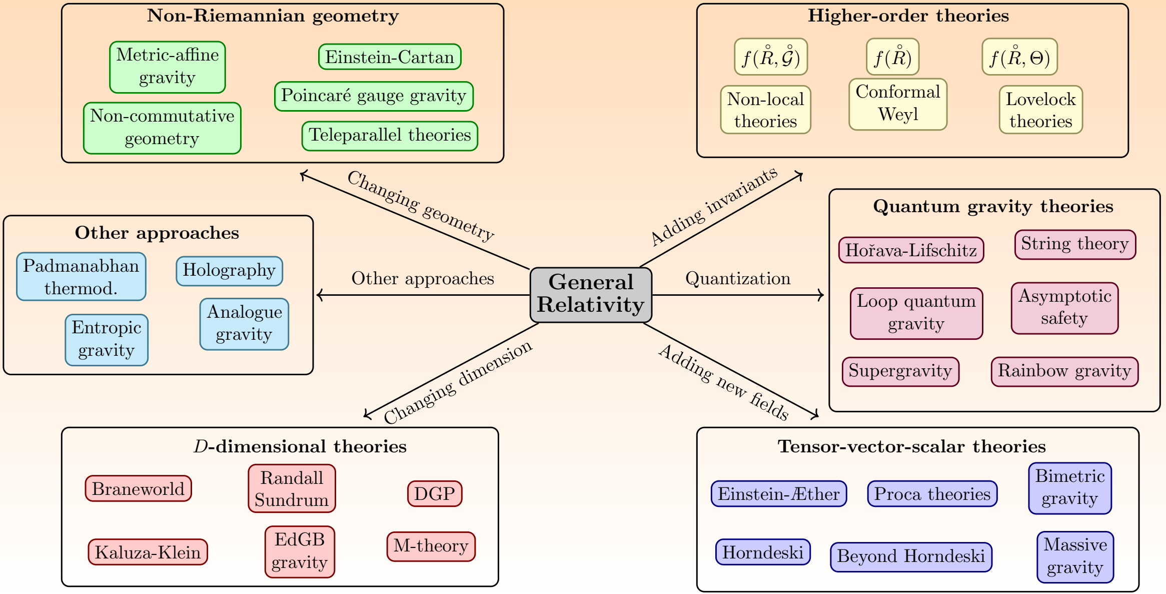
How to modify gravity?



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Why is geometry a well-motivated route?

General Relativity ties gravity to spacetime geometry, so changing the geometric structure is a direct way to explore new gravitational theories.

Modifying General Relativity from geometry

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Physical target

The goal is to modify GR from geometry and study the new effects for black holes, cosmology, or gravitational waves.

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- The physical question of this talk is:

can T or Q leave signatures in black holes and gravitational waves?

Curvature, torsion and nonmetricity

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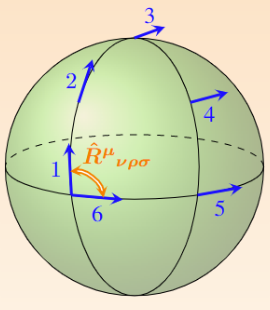
Curvature R



Directions change after transport around a loop.
96 independent components.

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = 2\partial_{[\mu}\tilde{\Gamma}^{\lambda}{}_{|\rho|\nu]} + 2\tilde{\Gamma}^{\lambda}{}_{\sigma[\mu}\tilde{\Gamma}^{\sigma}{}_{|\rho|\nu]}$$

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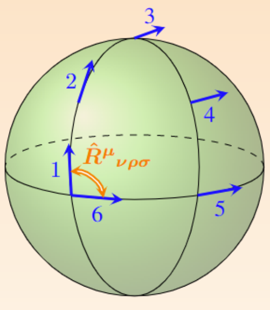

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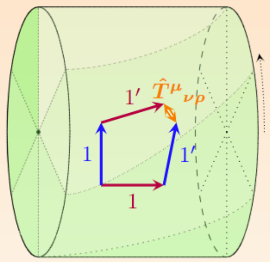

Infinitesimal parallelograms fail to close.
24 independent components.

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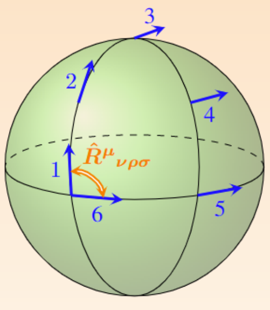

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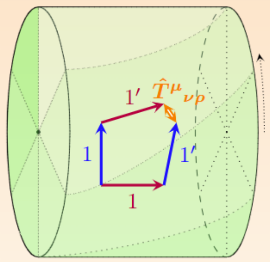

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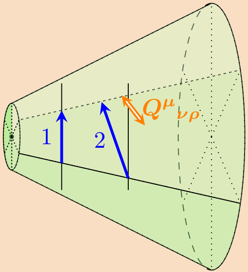
Nonmetricity Q



Lengths and angles change under transport.
40 independent components.
$Q_{\lambda\mu\nu} = \tilde{\nabla}_{\lambda} g_{\mu\nu}$

This talk focuses on geometries where R, T, and Q can participate in the dynamics.

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}\,,$$

with

$$N^{\lambda}{}_{\mu\nu} = K^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}\,,$$

where the contortion $K^{\lambda}{}_{\mu\nu}$ and disformation tensors $L^{\lambda}{}_{\mu\nu}$ are

$$K^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left(T^{\lambda}{}_{\mu\nu} - T_{\mu}{}^{\lambda}{}_{\nu} - T_{\nu}{}^{\lambda}{}_{\mu}\right)\,,$$

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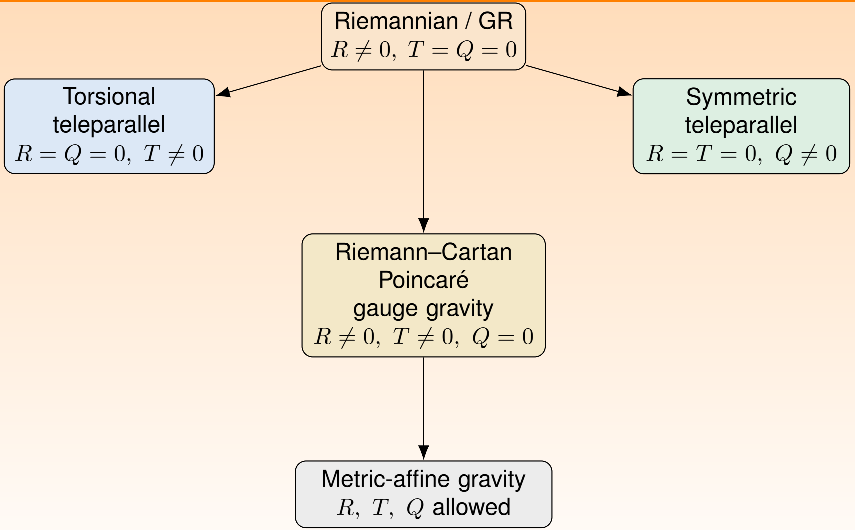
- General decomposition of the the curvature tensor:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|}N^{\lambda}{}_{\rho|\nu]} + 2N^{\lambda}{}_{\sigma[\mu|}N^{\sigma}{}_{\rho|\nu]}\,.$$

Different extended geometries give different types of gravitational theories



S. Bahamonde et al., Rep. Prog. Phys. 86, 026901 (2023)

Teleparallel theories: Trinity of Gravity

- Let us consider another particular interesting case known as ”Teleparallel Geometries” with the following condition:

$$\tilde{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \nabla_{\rho} N^{\mu}{}_{\nu\sigma} - \nabla_{\sigma} N^{\mu}{}_{\nu\rho} + N^{\mu}{}_{\tau\rho} N^{\tau}{}_{\nu\sigma} - N^{\mu}{}_{\tau\sigma} N^{\tau}{}_{\nu\rho} = 0\,.$$

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Ricci scalar decomposition

$$\tilde{R} = R + \Big(T + 2\mathring{\nabla}_{\mu}(\sqrt{-g}T^{\rho}{}_{\rho}{}^{\mu})\Big) + \Big(Q + \mathring{\nabla}_{\mu}Q^{\mu\nu}{}_{\nu} - \mathring{\nabla}_{\nu}Q_{\mu}{}^{\mu\nu}\Big) + C = 0$$

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with

$$T := T^{\rho\lambda\kappa}T_{\rho\lambda\kappa} + 2T^{\rho\lambda\kappa}T_{\kappa\rho\lambda} - 4T_{\rho}{}^{\kappa}{}_{\kappa}T^{\rho\lambda}{}_{\lambda}\,, \quad \text{Torsion scalar}\,,$$

$$Q := -\frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} + \frac{1}{2}Q_{\alpha\beta\gamma}Q^{\beta\alpha\gamma} + \frac{1}{4}Q_{\alpha}Q^{\alpha} - \frac{1}{2}Q_{\alpha}\bar{Q}^{\alpha}\,, \quad \text{Nonmetricity scalar}\,,$$

$$C := 2(Q_{\kappa\rho\lambda}T^{\lambda\kappa\rho} + Q_{\rho}{}^{\sigma}{}_{\sigma}T^{\rho\kappa}{}_{\kappa} - Q^{\sigma}{}_{\sigma\rho}T^{\rho\kappa}{}_{\kappa})\,.$$

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- Torsional teleparallel gravity, or metric teleparallelism, assumes $Q = \tilde{R} = 0$ and then $R = -T + B$ and then the TEGR:

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- Both theories are classically equivalent to GR.

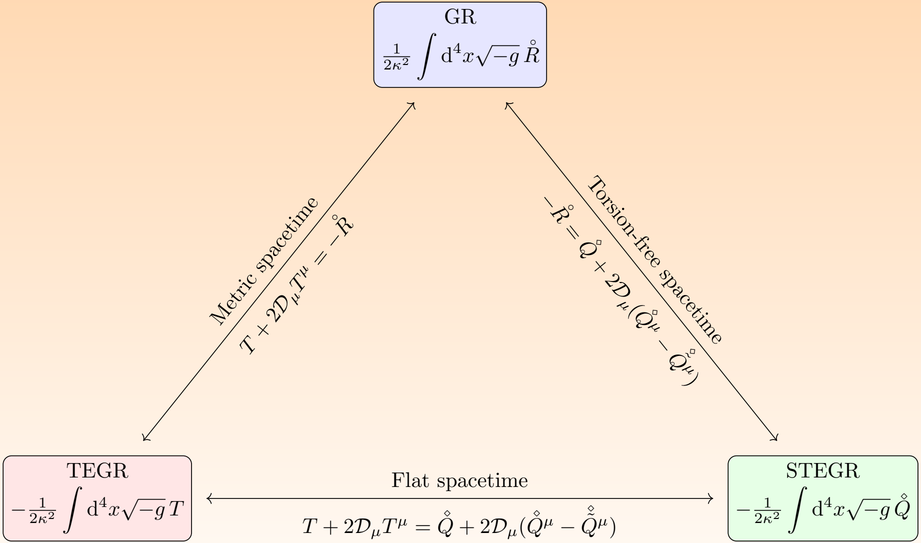

Figure: Geometrical trinity of gravity (S. Bahamonde et.al., "Teleparallel Gravity: From Theory to Cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.; J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, "The Geometrical Trinity of Gravity," Universe **5** (2019) no.7, 173.)

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- Therefore MAG allows, in general:

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So enlarging geometry is not just a mathematical generalisation: it changes both the physical content and the consistency conditions of the theory.

Quadratic Poincaré gauge theory - ghost issue

- The most general class of quadratic Poincaré gauge theory (parity preserving) is

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  2. Effectively 2 extra scalar fields+ torsional masses: (Holst)

$$\mathcal{L}_2 = \frac{M_{\rm pl}^2}{2}\tilde{R} + \alpha(\varepsilon_{\rho\lambda\mu\nu}\tilde{R}^{\rho\lambda\mu\nu})^2 + C_1 T_{\rho\mu\nu}T^{\rho\mu\nu} + C_2 T_{\rho\mu\nu}T^{\nu\rho\mu} + C_3 T^{\rho}{}_{\rho\mu}T^{\lambda}{}_{\lambda}{}^{\mu}$$

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The obstruction mainly applies to the most conservative quadratic Poincaré gauge setting. A healthier propagating torsion sector may still be obtained if one goes beyond these assumptions.

Relax some of the standard assumptions, for example:

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1. Break Poincaré gauge approach and consider torsion more like "EFT": $(\nabla T)^2, R\nabla T...$
2. Consider Cubic Poincaré gauge gravity. (cubic in field strength tensors)

Cubic Poincaré gauge theory

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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- Cubic parity preserving branch with mixing terms (26 h_i): (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **109** (2024) no.10, 10)

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- One can generalise it with nonmetricity as well (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) no.8, 084058)

Outline

Black-hole hair and spontaneous scalarization

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- In scalar-Gauss–Bonnet gravity, this instability is sourced by the Gauss–Bonnet invariant.
- In teleparallel scalar-Gauss–Bonnet gravity, we ask whether torsion can trigger the same phenomenon.

Teleparallel scalar Gauss–Bonnet: splitting the invariant

- In the usual Riemannian theory, spontaneous scalarization is driven by a coupling to the curvature Gauss–Bonnet invariant,

$$\mathcal{S}_{\mathrm{sGB}} = \frac{1}{2\kappa^2}\int\left[R - \frac{1}{2}\,\partial_\mu\psi\partial^\mu\psi + \mathcal{G}(\psi)\,G\right]\sqrt{-g}\,\mathrm{d}^4x\,, \quad G = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$$

S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107**, 104013 (2023).

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- The two teleparallel pieces are

$$\begin{aligned} T_G &= \delta^{\mu\nu\sigma\lambda}_{\alpha\beta\gamma\epsilon}K^{\alpha}{}_{\chi\mu}K^{\chi\beta}{}_{\nu}K^{\gamma}{}_{\xi\sigma}K^{\xi\epsilon}{}_{\lambda} + 2\delta^{\mu\nu\sigma\lambda}_{\alpha\beta\gamma\epsilon}K^{\alpha\beta}{}_{\mu}K^{\gamma}{}_{\chi\nu}K^{\chi\epsilon}{}_{\xi}K^{\xi}{}_{\sigma\lambda} \\ &\quad + 2\delta^{\mu\nu\sigma\lambda}_{\alpha\beta\gamma\epsilon}K^{\alpha\beta}{}_{\mu}K^{\gamma}{}_{\chi\nu}D_{\lambda}K^{\chi\epsilon}{}_{\sigma}\,, \\ B_G &= \frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\delta^{\mu\nu\sigma\lambda}_{\alpha\beta\gamma\epsilon}K^{\alpha\beta}{}_{\nu}\left(K^{\gamma}{}_{\xi\sigma}K^{\xi\epsilon}{}_{\lambda} - \frac{1}{2}R^{\gamma\epsilon}{}_{\sigma\lambda}\right)\right]. \end{aligned}$$

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- T_G is the teleparallel Gauss–Bonnet invariant, while B_G is the boundary contribution that completes the Riemannian invariant.

S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107**, 104013 (2023).

Teleparallel scalar Gauss–Bonnet: action and limits

- Since

$$G = T_G + B_G,$$

the scalar can couple independently to both teleparallel pieces:

$$\mathcal{S}_{\rm TsGB} = \frac{1}{2\kappa^2}\int \left[-T - \frac{1}{2}\beta\, \partial_\mu\psi\partial^\mu\psi + \alpha_1\mathcal{G}_1(\psi)T_G + \alpha_2\mathcal{G}_2(\psi)B_G\right]\sqrt{-g}\,\mathrm{d}^4x.$$

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- Three important limits are:
$$\alpha_3 = 0 \quad \Rightarrow \quad \text{standard Riemannian sGB},$$
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- The last two cases have no purely Riemannian analogue: scalarization is sourced by torsion.

Spontaneous scalarization in teleparallel sGB

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- A tachyonic scalar mode can appear when
$$\frac{-4\alpha_3\ddot{\mathcal{G}}_3(\psi_0) + 6\alpha_2\ddot{\mathcal{G}}_2(\psi_0) + 5\beta M^2}{20\beta M^3} < 0.$$

Spontaneous scalarization in teleparallel sGB

- We choose the coupling such that the GR branch remains a solution:

$$\psi = \psi_0, \qquad \mathcal{G}_i'(\psi_0) = 0.$$

- Perturbing the scalar field around Schwarzschild gives

$$\frac{d^2u}{dr_*^2} + \left[\omega^2 - U(r)\right] u = 0.$$

- The effective potential is

$$U(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{\ell(\ell+1)}{r^2} - \frac{32M}{r^5\beta}\alpha_3\ddot{\mathcal{G}}_3(\psi_0) + \frac{48M^2}{r^6\beta}\left(\alpha_3\ddot{\mathcal{G}}_3(\psi_0) + \alpha_2\ddot{\mathcal{G}}_2(\psi_0)\right)\right].$$

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- Compared with the Riemannian sGB case, the new α_3-term gives an additional torsional way to trigger scalarization.

Near-horizon expansion: regular scalarized black holes

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g_{tt} &= = a_1(r - r_H) + a_2(r - r_H)^2 + \cdots \,, \\
g_{rr}^{-1} &= = b_1(r - r_H) + b_2(r - r_H)^2 + \cdots \,, \\
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- We have two different branches. Branch I occurs when

$$\alpha_3 \dot{\mathcal{G}}_3(\psi_H) = \alpha_2 \dot{\mathcal{G}}_2(\psi_H),$$

and gives

$$\psi'_H = \frac{8\alpha_2 \dot{\mathcal{G}}_2(\psi_H)}{\beta r_H^3}.$$

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- This branch contains the standard Riemannian sGB condition as the limit $\alpha_3 = 0$:

$$\psi'_H = \frac{1}{4\alpha_2 r_H\dot{\mathcal{G}}_2}\left[r_H^2 \pm \sqrt{r_H^4 - \frac{96\alpha_2^2\dot{\mathcal{G}}_2^2}{\beta}}\right].$$

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- Thus teleparallel sGB changes not only the action, but also the regularity conditions selecting the scalarized branch.

Numerical setup: what is being tested?

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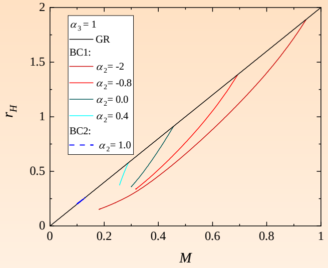
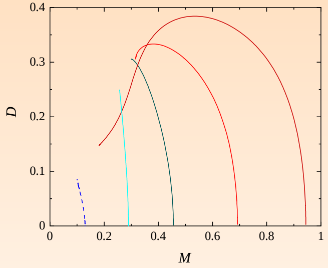
- The two purely teleparallel tests are

$$\alpha_2 = 0 \quad \Rightarrow \quad T_G\text{-driven scalarization},$$

$$\alpha_2\mathcal{G}_2 + \alpha_3\mathcal{G}_3 = 0 \quad \Rightarrow \quad B_G\text{-driven scalarization}.$$

Numerical result I: bifurcation and scalar charge





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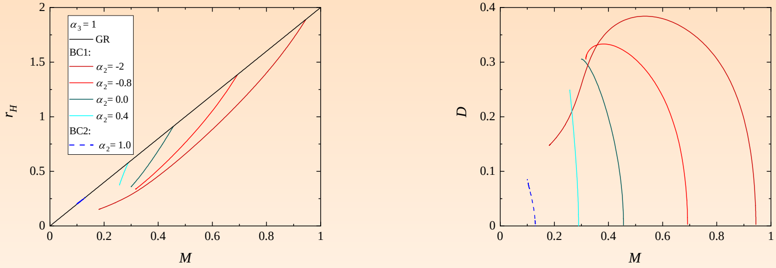

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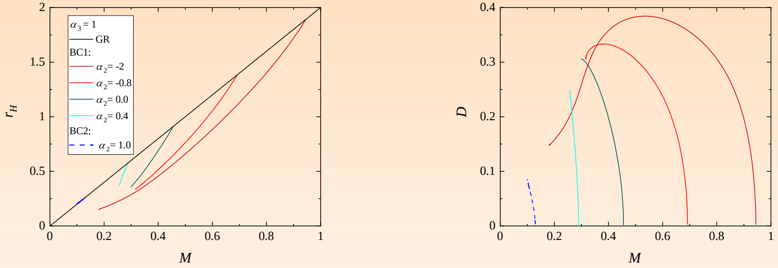

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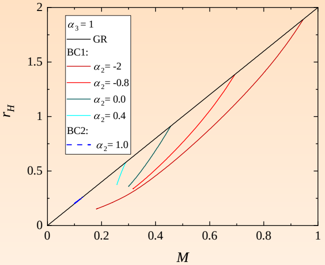

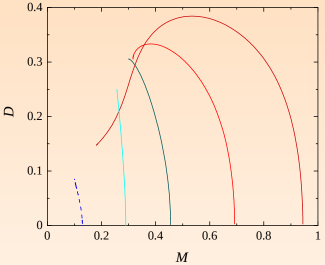

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- The scalar charge D also changes, showing that the asymptotic scalar hair is sensitive to the torsional Gauss–Bonnet sector.

Numerical result II: profiles beyond standard static sGB

S. Bahamonde, D. D. Doneva, L. Ducobu, C. Pfeifer and S. S. Yazadjiev, Phys. Rev. D **107**, 104013 (2023).

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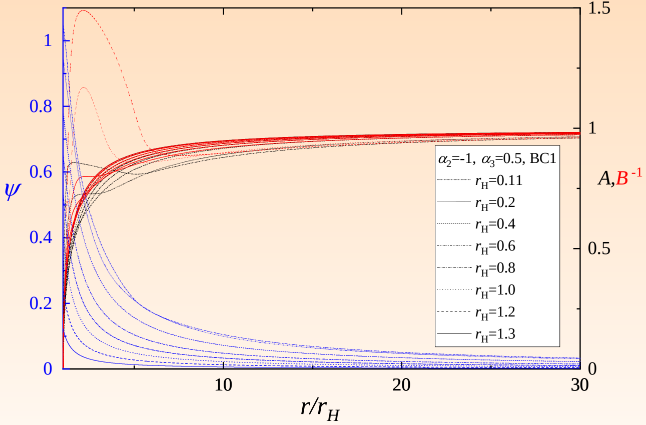

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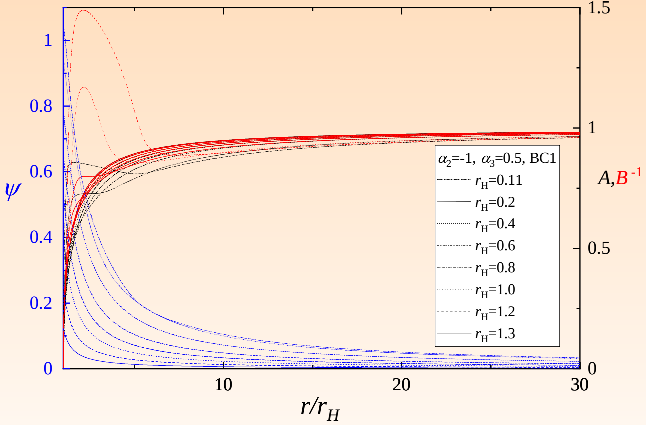

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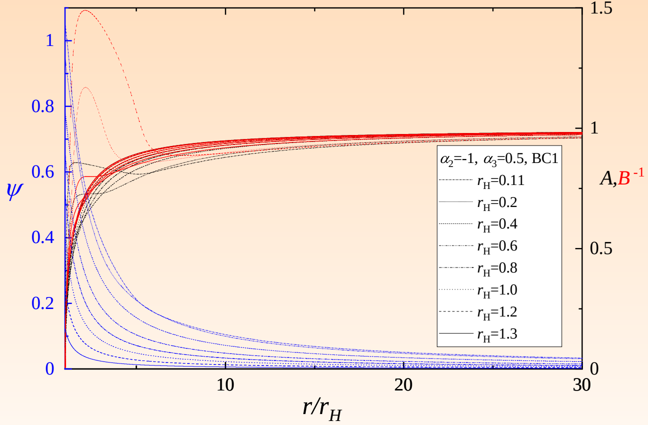

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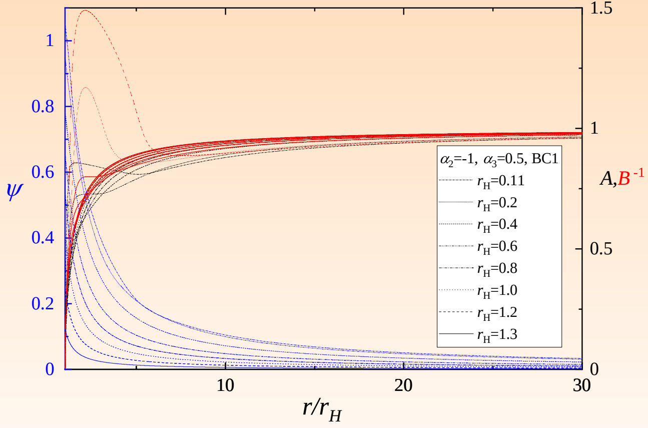

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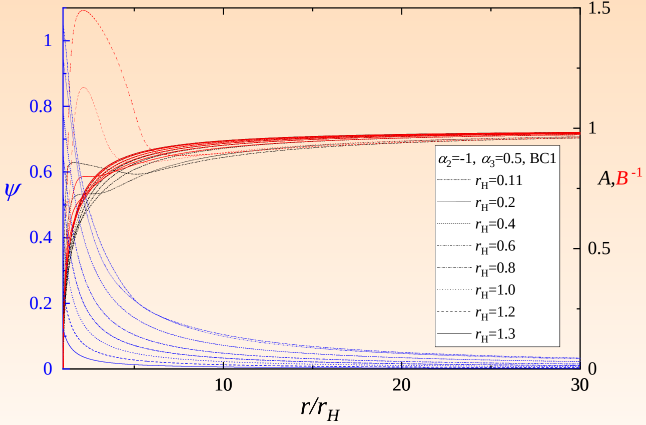

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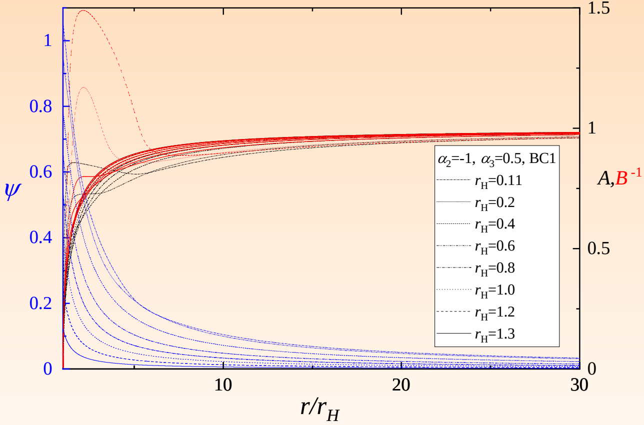

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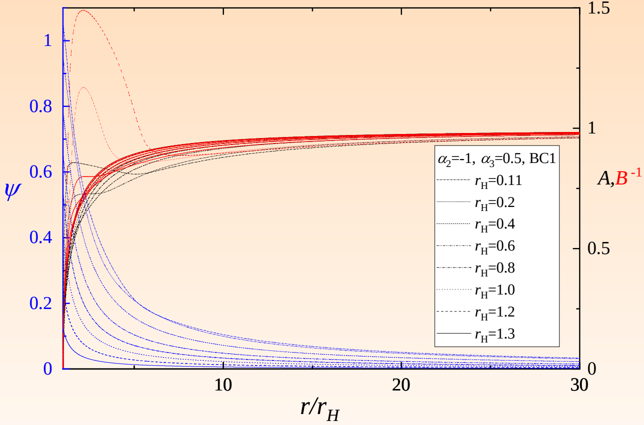

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Beyond the exponential coupling

- In a follow-up paper, we also studied monomial couplings,

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- Teleparallel contributions can modify the branch structure and may improve stability in some regions of parameter space.

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Physical lessons from teleparallel scalarization

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- The strong-field message is:

  teleparallel geometry changes black-hole scalarization, not just its notation.

Outline

1. Modifying gravity from geometry
2. Black holes in teleparallel gravity (zero curvature)
3. **Black holes in Metric-Affine theories (Non-zero curvature)**
4. Gravitational waves beyond Riemannian geometry

Spherically symmetric spacetimes in MAG (curvatureful case)

- Explicit symmetries on the metric and torsion tensors:

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- Static and spherically symmetric space-times:

$$\#10 \rightarrow \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right)\right. \;;$$

$$\#24 \rightarrow \#8 \left\{ \begin{array}{ccc} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{array} \right.$$

$$\#40 \rightarrow \#12 \left\{ \begin{array}{ccc} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{array} \right.$$

Electrodynamics coupled with torsion

- Now, consider another theory with couplings between the electromagnetic field strength $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $\tilde{R}^{\lambda}{}_{\rho\mu\nu}$:

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$$\Psi(r) = 1 - \frac{2m}{r} + \frac{1}{r^2}\left(k_1 q^2 - \frac{1}{2}k_3\kappa_{\rm s} q - \frac{1}{32k_2}k_3^2 q^2\right)\,, \quad A_\mu = \left(\frac{q}{r}, 0, 0, 0\right)\,.$$

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- New coupling between intrinsic spin charge κ_s and electric charge q.
- Different charges would give rise to different phenomenology. RN Cauchy problem can be evaded here!

Reissner–Nordström-like black holes

Exact static solution

In cubic metric-affine gravity with torsion and nonmetricity, one finds an exact static, spherically symmetric black-hole solution (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) 084058),

$$ds^2 = -\Psi(r)\,dt^2 + \frac{dr^2}{\Psi(r)} + r^2 d\Omega^2, \qquad \Psi(r) = 1 - \frac{2m}{r} + \frac{\mathcal{Q}_{\rm geom}}{r^2}.$$

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Why it is interesting

The metric looks like Reissner–Nordström, but the extra $1/r^2$ term is not an electric charge. It comes from intrinsic geometric charges carried by torsion and nonmetricity,

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Physical message

A familiar black-hole metric can therefore hide genuinely new geometric hair. For suitable signs of the constants H_i, one also finds a branch without an inner Cauchy horizon.

What are the intrinsic charges?

Matter can carry more than energy–momentum

In metric-affine gravity, matter may carry microstructure. In the black-hole solution, this appears through three intrinsic geometric charges:

$$\kappa_{\rm s} \text{ (spin)}, \qquad \kappa_{\rm d} \text{ (dilation)}, \qquad \kappa_{\rm sh} \text{ (shear)}.$$

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Why this matters

These charges survive in the vacuum exterior as integration constants of the geometry. GR has no direct analogue, because standard GR couples matter only through energy–momentum.

Geometric hair: physical interpretation

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- Therefore two black holes with similar metric profiles may have different microscopic geometric origin.
- This motivates looking at matter couplings and rotating solutions.

From atomic spin–orbit coupling to gravity

Atomic physics analogy

In atomic systems, orbital motion couples to intrinsic spin and produces a spin–orbit interaction,

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Why this is difficult

In Poincaré gravity, axial symmetry activates the full torsion sector, so finding an exact slowly rotating solution is already a highly non-trivial problem.

A new exact slowly rotating solution with axial torsion

Exact slowly rotating solution

We found an **exact slowly rotating Kerr-like black-hole solution** with non-trivial dynamical torsion:

$$ds^2 = \Psi(r)dt^2 - \frac{dr^2}{\Psi(r)} - r^2 d\vartheta^2 - r^2 \sin^2\vartheta\, d\phi^2 + 2a\big(1-\Psi(r)\big)\sin^2\vartheta\, dt\, d\phi, \qquad \Psi(r) = 1 - \frac{2m}{r}.$$

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New torsion-induced interaction

Schematically, the solution contains

$$\mathcal{L}_{\rm eff} = \underbrace{\frac{d_1 N_1^2 \kappa_{\rm s}^2}{8\pi r^4}}_{\text{static spin charge}} + \underbrace{\frac{d_1 N_1 a \kappa_{\rm s}}{2\pi} F(r,\vartheta)}_{\text{new } a\kappa_{\rm s} \text{ term}}.$$

Here the same function $F(r,\vartheta)$ that appears in the axial torsion controls the new interaction.

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The key new effect is a purely torsion-induced coupling between black-hole rotation a and intrinsic spin charge κ_{s}: a gravitational spin–orbit interaction.

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Energy extraction without torsion: the Kerr reference case

- For a rotating black hole, the near-horizon frequency is shifted by the horizon angular velocity:

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- Therefore, without torsion, fermions do not give the usual bosonic amplification mechanism.
- The question is whether axial torsion can modify the near-horizon fermionic energy balance.

Dirac fermion with axial torsion

Adding torsion to the fermionic problem

To test whether torsion can modify the Kerr energy balance, we now consider a minimally coupled Dirac field. Only the axial torsion mode enters:

$$\gamma^\mu \nabla_\mu \psi - \frac{i}{4}\gamma^5 \gamma^\mu S_\mu \psi + i\mu\psi = 0.$$

Thus fermions probe S_μ, not the full torsion tensor.

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Near-horizon data

Requiring separability fixes the angular structure of the axial mode,

$$F(r,\vartheta) = \frac{3 - 4\Psi(r)}{6r\Psi(r)}\cos\vartheta + \frac{f(r)}{r}.$$

The near-horizon splitting is therefore controlled by

$$\kappa_{\rm s}, \qquad f(r_h).$$

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Chiral splitting and energy extraction

Near-horizon frequencies

$$\Omega_{\pm} = \omega - \frac{ak}{2mr_h} \pm \frac{3}{r_h}\left(N_1\kappa_{\rm s} - \frac{a}{m}f(r_h)\right).$$

ω: mode frequency, k: azimuthal number, a: black-hole rotation, m: mass, r_h: horizon radius.
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Here A_2 and A_3 are the amplitudes of the two ingoing helicity components. The number flux remains positive,

$$\frac{dN}{dt} \approx \frac{1}{4}\left(|A_3|^2 + |A_2|^2\right) > 0.$$

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Axial torsion opens a new possibility for black-hole energy extraction, even without standard fermionic

Gravitational waves: the GR reference point

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- These polarizations describe how a ring of freely falling particles is distorted by the wave.

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- Far from the source, GR has two tensor polarizations:

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Outline

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- The question is whether these modes propagate consistently and whether they leave observable signatures.

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- The main gravitational-wave message is simple:

extra geometry $\Rightarrow$ extra possible wave channels.

Teleparallel Horndeski and GW polarizations

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- Thus GW polarizations are a natural observational window into teleparallel modifications of gravity.

Exact pp-waves in cubic metric-affine gravity

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$$t_{22} = \text{torsion mode}, \qquad w = \text{Weyl-vector mode}, \qquad \lambda = \text{traceless nonmetricity mode}.$$
- Therefore the wave is not only a metric wave: torsion and nonmetricity also carry part of the exact non-linear gravitational wave.

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- For an observer, this means that the affine sector can appear as an additional breathing-type response beyond the usual $+$ and $\times$ tensor modes.
- The important point is not only that the metric wave profile changes, but that the affine sector can leave an independent polarization imprint.

Main conclusions

- Enlarging the geometry of spacetime gives a controlled way to go beyond GR:

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- Gravitational waves provide another probe of non-Riemannian geometry, through extra wave channels and exact pp-wave solutions with torsion and nonmetricity.


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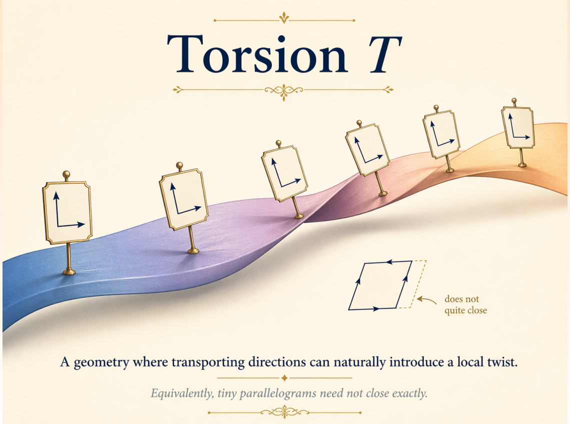

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