Rethinking Gravity: From Torsion and Nonmetricity to Black Holes and Cosmology

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USTC seminar, 12/May/2026

Outline

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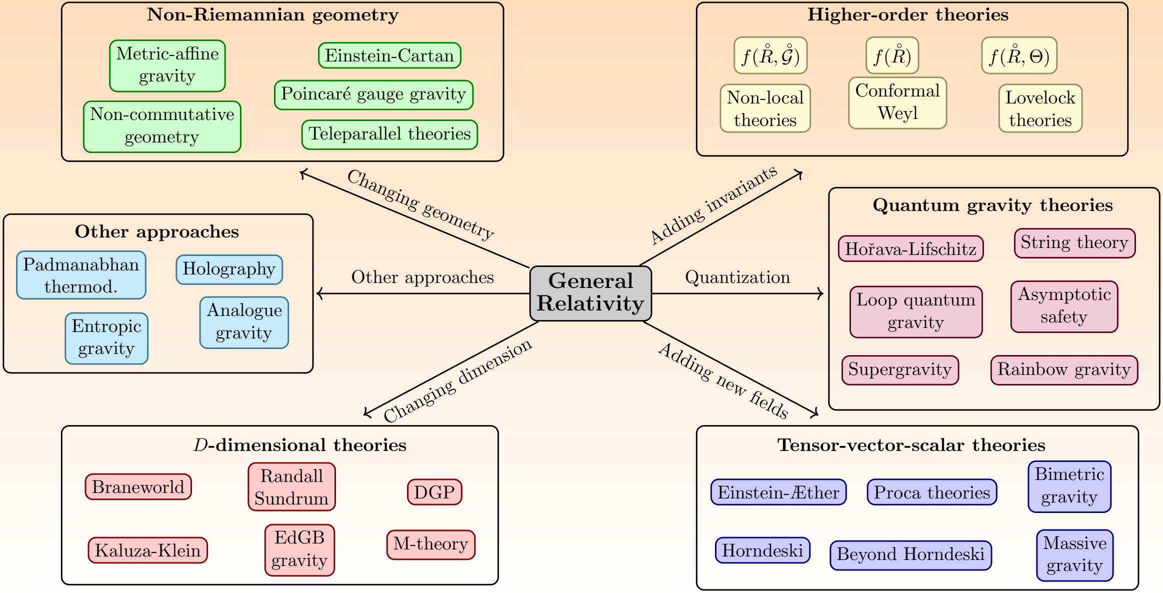
How to modify gravity?



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Why is geometry a well-motivated route?

General Relativity ties gravity to spacetime geometry, so changing the geometric structure is a direct way to explore new gravitational theories.

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Physical target

The goal is to modify GR from geometry and study the new effects for black holes, cosmology, or gravitational waves.

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  - The general curvature is defined as in GR but changing $\Gamma^{\lambda}{}_{\mu\nu}$ to $\tilde{\Gamma}^{\lambda}{}_{\mu\nu}$:

$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = \partial_{\mu}\tilde{\Gamma}^{\lambda}{}_{\rho\nu} - \partial_{\nu}\tilde{\Gamma}^{\lambda}{}_{\rho\mu} + \tilde{\Gamma}^{\lambda}{}_{\sigma\mu}\tilde{\Gamma}^{\sigma}{}_{\rho\nu} - \tilde{\Gamma}^{\lambda}{}_{\sigma\nu}\tilde{\Gamma}^{\sigma}{}_{\rho\mu}$$

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- MAG extends GR to include more general geometric structures, enabling richer interactions with matter (e.g., spin, microstructure).

Curvature, torsion and nonmetricity

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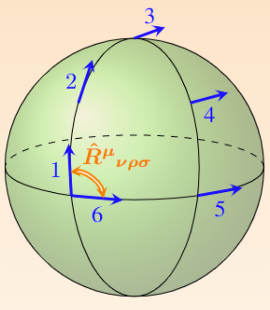
Curvature R



Directions change after transport around a loop.
96 independent components.

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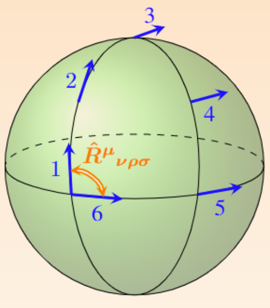

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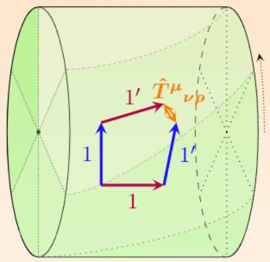
Torsion T



Infinitesimal parallelograms fail to close.
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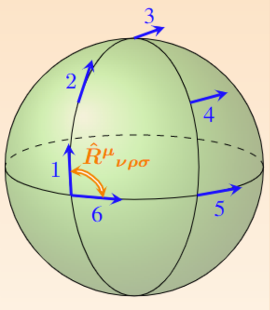

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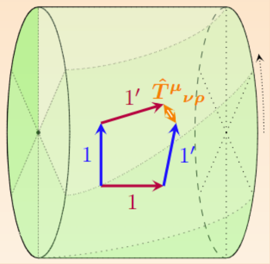

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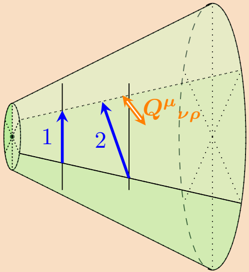
Nonmetricity Q



Lengths and angles change under transport.
40 independent components.
$Q_{\lambda\mu\nu} = \tilde{\nabla}_{\lambda} g_{\mu\nu}$

This talk focuses on geometries where R, T, and Q can participate in the dynamics.

Post-Riemannian decomposition

- It is useful to separate the connection as

$$\tilde{\Gamma}^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} + N^{\lambda}{}_{\mu\nu}\,,$$

with

$$N^{\lambda}{}_{\mu\nu} = K^{\lambda}{}_{\mu\nu} + L^{\lambda}{}_{\mu\nu}\,,$$

where the contortion $K^{\lambda}{}_{\mu\nu}$ and disformation tensors $L^{\lambda}{}_{\mu\nu}$ are

$$K^{\lambda}{}_{\mu\nu} = \frac{1}{2}\left(T^{\lambda}{}_{\mu\nu} - T_{\mu}{}^{\lambda}{}_{\nu} - T_{\nu}{}^{\lambda}{}_{\mu}\right)\,,$$

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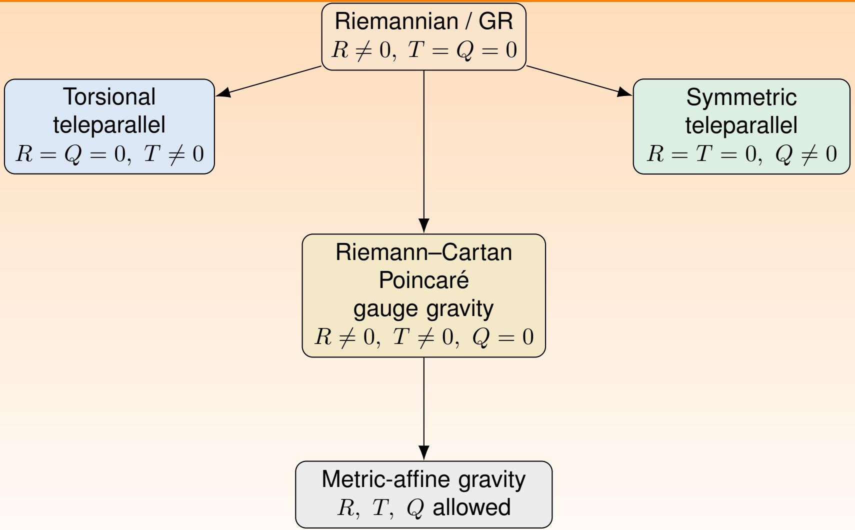
- General decomposition of the the curvature tensor:
$$\tilde{R}^{\lambda}{}_{\rho\mu\nu} = R^{\lambda}{}_{\rho\mu\nu} + 2\nabla_{[\mu|}N^{\lambda}{}_{\rho|\nu]} + 2N^{\lambda}{}_{\sigma[\mu|}N^{\sigma}{}_{\rho|\nu]}\,.$$

Different extended geometries give different types of gravitational theories



S. Bahamonde et al., Rep. Prog. Phys. 86, 026901 (2023)

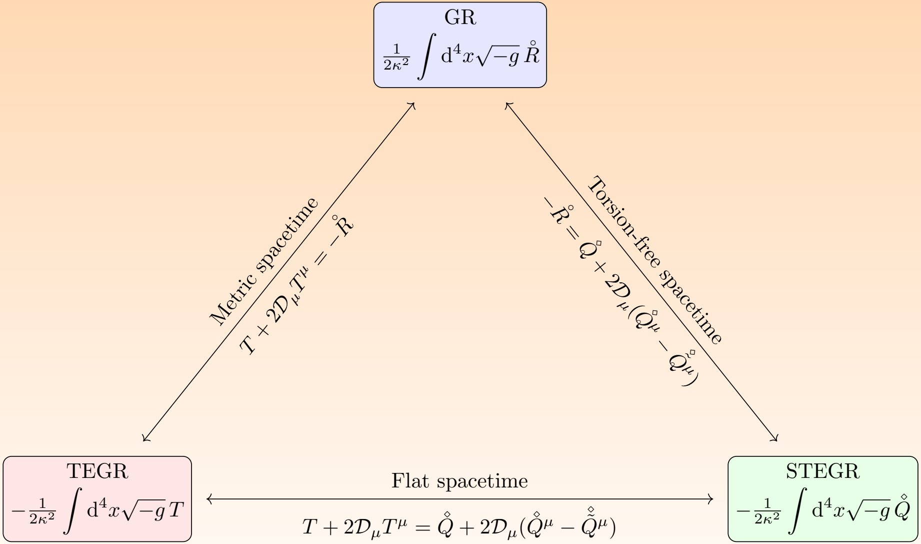


Figure: Geometrical trinity of gravity (S. Bahamonde et.al., "Teleparallel Gravity: From Theory to Cosmology," Rept. Prog. Phys. **86** (2023) no.2, 026901.; J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, "The Geometrical Trinity of Gravity," Universe **5** (2019) no.7, 173.)

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- Therefore MAG allows, in general:

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So enlarging geometry is not just a mathematical generalisation: it changes both the physical content and the consistency conditions of the theory.

Dynamics in MAG gauge theories

- Gravitational action with dynamical torsion and nonmetricity

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3. **Intrinsic Shears** term ${}^{(\mathrm{sh})}\Delta_{(\mu\nu)\lambda}$: source of **traceless nonmetricity**

Quadratic Poincaré gauge theory - ghost issue

- The most general class of quadratic Poincaré gauge theory (parity preserving) is

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$$\mathcal{L}_2 = \frac{M_{\rm pl}^2}{2}\tilde{R} + \alpha(\varepsilon_{\rho\lambda\mu\nu}\tilde{R}^{\rho\lambda\mu\nu})^2 + C_1 T_{\rho\mu\nu}T^{\rho\mu\nu} + C_2 T_{\rho\mu\nu}T^{\nu\rho\mu} + C_3 T^{\rho}{}_{\rho\mu}T^{\lambda}{}_{\lambda}{}^{\mu}$$

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The obstruction mainly applies to the most conservative quadratic Poincaré gauge setting. A healthier propagating torsion sector may still be obtained if one goes beyond these assumptions.

Relax some of the standard assumptions, for example:

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2. Consider Cubic Poincaré gauge gravity. (cubic in field strength tensors)

Cubic Poincaré gauge theory

- Convenient to decompose torsion as

$$T^{\lambda}{}_{\mu\nu} = \frac{1}{3}\left(\delta^{\lambda}{}_{\nu}T_{\mu} - \delta^{\lambda}{}_{\mu}T_{\nu}\right) + \frac{1}{6}\varepsilon^{\lambda}{}_{\rho\mu\nu}S^{\rho} + t^{\lambda}{}_{\mu\nu}\,.$$

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- We showed that by including these Poincaré gauge invariants, ghost issue in the vector sector is solved!
- One can generalise it with nonmetricity as well (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) no.8, 084058)

Outline

Brief review: Riemann–Cartan background cosmology

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- These torsional background modes can modify the Friedmann equations and have been used to model:
  - non-singular bouncing cosmologies;
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  - inflationary scenarios, including Einstein–Cartan models coupled to the Higgs sector.
- The next step is to understand whether these backgrounds remain viable once perturbations of the metric and connection are included.

Cosmology in MAG

- Let us assume that the metric, torsion and nonmetricity have the same cosmological symmetries (isotropy and homogeneity)

$$\mathcal{L}_\xi \bar{g}_{\mu\nu} = \mathcal{L}_\xi \bar{T}^\lambda{}_{\mu\nu} = \mathcal{L}_\xi \bar{Q}^\lambda{}_{\mu\nu} = 0 \,.$$

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- Solving these symmetry conditions gives the FLRW metric and the most general homogeneous and isotropic torsion and nonmetricity tensors:

$$\begin{aligned}
\bar{g} &= -\bar{n}_\mu \bar{n}_\nu \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu + \bar{P}_{\mu\nu} \mathrm{d}x^\mu \otimes \mathrm{d}x^\nu = -N^2 \mathrm{d}t \otimes \mathrm{d}t + a^2 \gamma_{ij} \mathrm{d}x^i \otimes \mathrm{d}x^j\,,\\
\bar{T}^\lambda{}_{\mu\nu} &= 2T_1(t)\, \bar{n}_{[\mu} \bar{P}_{\nu]}{}^\lambda + 2T_2(t)\, \bar{\varepsilon}^\lambda{}_{\mu\nu\rho} \bar{n}^\rho\,,\\
\bar{Q}_{\lambda\mu\nu} &= 2Q_1(t) \bar{n}_\lambda \bar{n}_\mu \bar{n}_\nu + 2Q_2(t) \bar{n}_\lambda \bar{P}_{\mu\nu} + 2Q_3(t) \bar{P}_{\lambda(\mu} \bar{n}_{\nu)}\,,
\end{aligned}$$

where $\gamma_{ij} \mathrm{d}x^i \otimes \mathrm{d}x^j = \frac{\mathrm{d}r^2}{1-Kr^2} + r^2 \mathrm{d}\Omega^2$. Note that there are 5 independent functions coming from Post-Riemannian.

- One can consider that the energy-momentum tensor is described by a standard perfect fluid described by

$$T_{\mu\nu} = \left(\rho(t) + p(t)\right) n_\mu n_\nu + p(t) g_{\mu\nu} = \rho(t) n_\mu n_\nu + p(t) p_{\mu\nu}\,,$$

Cosmology in MAG - matter

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- Considering matter described by an unconstrained hyperfluid respecting the cosmological principle (isotropy and homogeneity), we find that the hypermomentum is

$$\Delta_{\lambda\mu\nu} = \frac{1}{3}\Delta_1(t) p_{\lambda\mu} n_\nu + \Delta_2(t) p_{\lambda\nu} n_\mu + \Delta_3(t) n_\lambda p_{\mu\nu} + \frac{1}{4}\Delta_4(t) n_\lambda n_\mu n_\nu + \Delta_5(t) \varepsilon_{\lambda\mu\nu\rho} n^\rho \,,$$

which contains 5 independent source functions associated with intrinsic spin, dilation, and shear hypermomentum.

Background Cosmology in Poincaré Gauge Gravity

- By imposing that the matter sector respects diffeomorphism invariance, we arrive at the following generalised conservation equation ($Q = 0$)

$$\sqrt{-g}(2\nabla_\mu T^\mu{}_\alpha - \Delta^{\lambda\mu\nu}\tilde{R}_{\lambda\mu\nu\alpha}) + \tilde{\nabla}_\mu\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\alpha{}^{\mu\nu}) + 2T_{\mu\alpha}{}^\lambda\tilde{\nabla}_\nu(\sqrt{-g}\Delta_\lambda{}^{\mu\nu}) = 0\,.$$

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- When Nonmetricity is vanishing, only intrinsic spin contributes

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- In our Cubic Poincaré Gauge Gravity theory, we have the modified FLRW equations of the form:

S. Bahamonde, R. Briffa, K. Dialektopoulos, D. Iosifidis and J. Levi Said, Phys. Dark Univ. **52** (2026), 102249.

$$3H^2 + f(T_1(t),T_2(t)) \;=\; \kappa^2\rho\,, \quad 3H^2 + 2\dot{H} + g(T_1(t),T_2(t)) = -\kappa^2 p\,.$$

and the connection equations:

$$\begin{aligned} 2\kappa^2\,{}^{(\mathrm{s})}\Delta_3 &= 3T_1\Big[6h_1(H-T_1)(H-2T_1) - 6(h_1-16h_{13})T_2^2 + m_T^2\Big]\,, \\ \kappa^2\,{}^{(\mathrm{s})}\Delta_5 &= 3T_2\Big[48h_{13}\left(-H^2+T_1^2+2T_2^2\right) - 3h_1T_1^2 + 8m_S^2\Big]\,. \end{aligned}$$

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- The theory at the background level depends on h_1, h_{13} and the mass parameters m_S, m_T.

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$$\dot{\rho} + 3H(\rho + p) = 3\,{}^{(\mathrm{s})}\Delta_5\left(HT_2 + \dot{T}_2\right) - 3\,{}^{(\mathrm{s})}\Delta_3\left(\dot{H} - HT_1 + H^2 - \dot{T}_1\right)\,.$$

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- C_1 and C_3 modify the gravitational coupling, C_2 is an effective 'spatial curvature' term and C_4 alters the radiation energy density, all coming due to the presence of hypermomentum.
- A flat FLRW geometry produces the same term as a nonflat geometry with hypermomentum playing this role.

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- In metric-affine gravity the connection also fluctuates, so torsion and nonmetricity add new helicity sectors beyond the metric ones.
- Hence SVT tells us which physical sectors can propagate independently on FLRW.

SVT decomposition around FLRW: geometric sectors

- Including parity, the geometric perturbations organise as

$$\delta g_{\mu\nu} \longrightarrow 0^+ \oplus 1^+ \oplus 2^+,$$
$$\delta T^{\lambda}{}_{\mu\nu} \longrightarrow 0^+ \oplus 0^- \oplus 1^+ \oplus 1^- \oplus 2^+ \oplus 2^-,$$
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- In the following, I focus on

$$2^+ \oplus 2^- \text{ torsion} \quad \Rightarrow \quad \text{extra massive spin-2 mode.}$$

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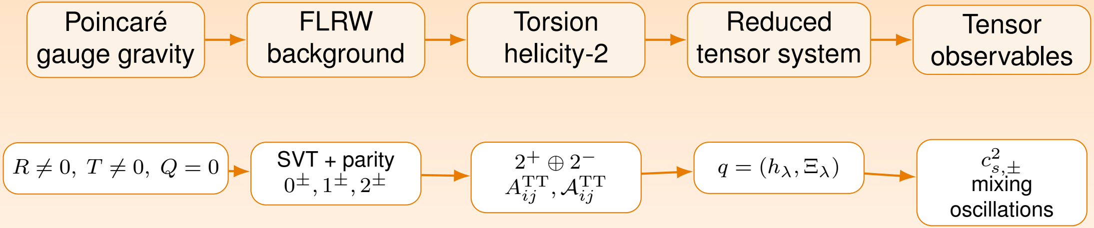
From torsion to tensor observables



From the full torsional geometry, I isolate the helicity-2 torsion sector and study how it mixes with the usual graviton on FLRW backgrounds.

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We assume a torsion-free background, but allow torsion to propagate at the perturbative level. In this sector, the relevant torsion piece is $t_{\rho\mu\nu}$. The post-Riemannian form of the theory is

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}R + \frac{\alpha}{2}R^2 + \frac{\beta}{2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} + 2\beta R_{\mu\nu}\nabla_\rho t^{\mu\nu\rho} + \beta\nabla_\rho t^{\mu\nu\rho}\nabla^\sigma t_{\mu\nu\sigma} + (q_5 + \frac{1}{2}q_6)\nabla_\mu t^{\mu\nu\lambda}\nabla_\rho t^\rho{}_{\nu\lambda}$$
$$+ \frac{1}{2}\gamma\, {M_T}^2 t_{\mu\nu\rho}t^{\mu\nu\rho} - 2(b_1-\beta)R^{\mu\rho\lambda\sigma}t_\mu{}^\nu{}_\lambda t_{\rho\nu\sigma} - 4(\beta-b_2)R^{\mu\sigma\lambda\rho}t_\mu{}^\nu{}_\lambda t_{\rho\nu\sigma} - 2(\beta-b_3)R_{\mu\nu}t_\lambda{}^\nu{}_\rho t^{\lambda\mu\rho}$$
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Around Minkowski, the torsional sector mixes with the Weyl-squared spin-2 mode and can remove the usual massive spin-2 ghost, leaving an additional healthy massive spin-2 excitation.

Quadratic Action for helicity-2 sector

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$$\mathcal{L}_T = f_R \left(h'_{ij}h'^{ij} - \partial_k h_{ij}\partial^k h^{ij}\right) + \beta\big(X'_{ij} - \frac{1}{2}D_- h_{ij}\big)^2 - X_{ij}X^{ij}\big(f_T + (c_1 - c_3 - \beta)\mathcal{H}' + (c_3 - \beta)\mathcal{H}^2\big)$$
$$+ \beta\mathcal{H}X^{ij}(D_+ h_{ij}) + \partial^2 Y^{ij}\left[\beta\left(\frac{1}{4}\partial^2 Y_{ij} + \frac{1}{2}Dh_{ij} - \frac{1}{a}(aX_{ij})'\right) - \frac{1}{4}Y_{ij}\big(f_T + (c_1 - c_2 - 2\beta)\mathcal{H}' + c_2\mathcal{H}^2\big)\right].$$

with

$$A_{ij} = \frac{X_{ij}}{a}, \qquad \mathcal{A}_{ij} = \frac{1}{a}\epsilon_{kl(i}\partial^k Y_{j)}{}^l,$$
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- No Ostrogradsky ghosts in Minkowski and additional massive spin-2.

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$$D_{-}h_{ij} = h''_{ij} - \partial^2 h_{ij}.$$

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and impose this relation with a Lagrange multiplier Ξ_{ij}:

$$\begin{aligned} S_T &\equiv S_T|_{h''=\Phi} + \int d\tau d^3x \, \left(\Phi^{ij} - h''^{ij}\right) \Xi_{ij} \\ &= S_T|_{h''=\Phi} + \int d\tau d^3x \, \left(\Phi^{ij}\Xi_{ij} + h'^{ij}\Xi'_{ij}\right) + \text{b.t.} \end{aligned}$$

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- The equation for Φ_{ij} is algebraic, so Φ_{ij} can be solved and substituted back.

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- The matrices $\mathcal{K}$, $\mathcal{M}$, and $\mathcal{V}$ control respectively ghosts, friction/mixing, and gradient/mass stability.

Equations of motion and physical interpretation

- The equations of motion take the matrix form

$$\mathcal{K}q'' + \left(\mathcal{M} - \mathcal{M}^T + \mathcal{K}'\right) q' + \left(\mathcal{V} + \mathcal{M}'\right) q = 0.$$

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- In FLRW, the cosmological background mixes them.

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- The theory can be stable, but tensor propagation is very different from GR.

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Garriga, Gorji, Hajkarim, Sasaki, 2508.08481

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What effects can this extra spin-2 mode produce?

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- This can produce oscillatory or scale-dependent features in the tensor power spectrum.

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Outline

Spherically symmetric spacetimes

- Explicit symmetries on the metric and torsion tensors:

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{L}_\xi T^{\lambda}{}_{\mu\nu} = \mathcal{L}_\xi Q_{\lambda\mu\nu} = 0 \implies \mathcal{L}_\xi \tilde{R}^{\lambda}{}_{\rho\mu\nu} = 0\,.$$

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- Static and spherically symmetric space-times:

$$\#10 \to \#2 \left\{ ds^2 = \Psi_1(r)\,dt^2 - \frac{dr^2}{\Psi_2(r)} - r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \right. ;$$

$$\#24 \to \#8 \left\{ \begin{matrix} T^t{}_{tr} & T^r{}_{tr} & T^\vartheta{}_{t\vartheta} \\ T^\vartheta{}_{r\vartheta} & T^\vartheta{}_{t\varphi} & T^\vartheta{}_{r\varphi} \\ T^t{}_{\vartheta\varphi} & T^r{}_{\vartheta\varphi} & \end{matrix} \right.$$

$$\#40 \to \#12 \left\{ \begin{matrix} Q_{ttt} & Q_{trr} & Q_{ttr} \\ Q_{t\vartheta\vartheta} & Q_{rtt} & Q_{rrr} \\ Q_{rtr} & Q_{r\vartheta\vartheta} & Q_{\vartheta t\vartheta} \\ Q_{\vartheta r\vartheta} & Q_{\vartheta t\varphi} & Q_{\vartheta r\varphi} \end{matrix} \right.$$

Reissner–Nordström-like black holes

Exact static solution

In cubic metric-affine gravity with torsion and nonmetricity, one finds an exact static, spherically symmetric black-hole solution (S. Bahamonde and J. Gigante Valcarcel, Phys. Rev. D **111** (2025) 084058),

$$ds^2 = -\Psi(r)\, dt^2 + \frac{dr^2}{\Psi(r)} + r^2 d\Omega^2, \qquad \Psi(r) = 1 - \frac{2m}{r} + \frac{\mathcal{Q}_{\rm geom}}{r^2}.$$

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Why it is interesting

The metric looks like Reissner–Nordström, but the extra $1/r^2$ term is not an electric charge. It comes from intrinsic geometric charges carried by torsion and nonmetricity,

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Physical message

A familiar black-hole metric can therefore hide genuinely new geometric hair. For suitable signs of the constants H_i, one also finds a branch without an inner Cauchy horizon.

What are the intrinsic charges?

Matter can carry more than energy–momentum

In metric-affine gravity, matter may carry microstructure. In the black-hole solution, this appears through three intrinsic geometric charges:

$$\kappa_{\rm s}\ \text{(spin)}, \qquad \kappa_{\rm d}\ \text{(dilation)}, \qquad \kappa_{\rm sh}\ \text{(shear)}.$$

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Why this matters

These charges survive in the vacuum exterior as integration constants of the geometry. GR has no direct analogue, because standard GR couples matter only through energy–momentum.

From atomic spin–orbit coupling to gravity

Atomic physics analogy

In atomic systems, orbital motion couples to intrinsic spin and produces a spin–orbit interaction,

$$\mathcal{L}_{\rm SO} \sim \lambda(r)\,\mathbf{L}\cdot\mathbf{S}.$$

This is one of the standard mechanisms behind energy-level splitting.

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Why this is difficult

In Poincaré gravity, axial symmetry activates the full torsion sector, so finding an exact slowly rotating solution is already a highly non-trivial problem.

A new exact slowly rotating solution with axial torsion

Exact slowly rotating solution

We found an **exact slowly rotating Kerr-like black-hole solution** with non-trivial dynamical torsion:

$$ds^2 = \Psi(r)dt^2 - \frac{dr^2}{\Psi(r)} - r^2 d\vartheta^2 - r^2 \sin^2\vartheta\, d\phi^2 + 2a\big(1-\Psi(r)\big)\sin^2\vartheta\, dt\, d\phi, \qquad \Psi(r) = 1 - \frac{2m}{r}.$$

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New torsion-induced interaction

Schematically, the solution contains

$$\mathcal{L}_{\rm eff} = \underbrace{\frac{d_1 N_1^2 \kappa_{\rm s}^2}{8\pi r^4}}_{\text{static spin charge}} + \underbrace{\frac{d_1 N_1 a \kappa_{\rm s}}{2\pi} F(r,\vartheta)}_{\text{new } a\kappa_{\rm s} \text{ term}}.$$

Here the same function $F(r,\vartheta)$ that appears in the axial torsion controls the new interaction.

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The key new effect is a purely torsion-induced coupling between black-hole rotation a and intrinsic spin charge κ_{s}: a gravitational spin–orbit interaction.

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Energy extraction without torsion: the Kerr reference case

- For a rotating black hole, the near-horizon frequency is shifted by the horizon angular velocity:

$$\tilde{\omega} = \omega - k\Omega_H , \qquad \Omega_H \simeq \frac{a}{2mr_h} .$$

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- Therefore, without torsion, fermions do not give the usual bosonic amplification mechanism.
- The question is whether axial torsion can modify the near-horizon fermionic energy balance.

Dirac fermion with axial torsion

Adding torsion to the fermionic problem

To test whether torsion can modify the Kerr energy balance, we now consider a minimally coupled Dirac field. Only the axial torsion mode enters:

$$\gamma^{\mu}\nabla_{\mu}\psi - \frac{i}{4}\gamma^{5}\gamma^{\mu}S_{\mu}\psi + i\mu\psi = 0.$$

Thus fermions probe S_{μ}, not the full torsion tensor.

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Near-horizon data

Requiring separability fixes the angular structure of the axial mode,

$$F(r,\vartheta) = \frac{3 - 4\Psi(r)}{6r\Psi(r)} \cos\vartheta + \frac{f(r)}{r}.$$

The near-horizon splitting is therefore controlled by

$$\kappa_{\mathrm{s}}, \qquad f(r_h).$$

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Chiral splitting and energy extraction

Near-horizon frequencies

$$\Omega_{\pm} = \omega - \frac{ak}{2mr_h} \pm \frac{3}{r_h}\left(N_1\kappa_{\rm s} - \frac{a}{m}f(r_h)\right).$$

ω: mode frequency, k: azimuthal number, a: black-hole rotation, m: mass, r_h: horizon radius.
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Here A_2 and A_3 are the amplitudes of the two ingoing helicity components. The number flux remains positive,

$$\frac{dN}{dt} \approx \frac{1}{4}\left(|A_3|^2 + |A_2|^2\right) > 0.$$

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Axial torsion opens a new possibility for black-hole energy extraction, even without standard fermionic

Main conclusions

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- Rotation plus axial torsion gives a new spin–orbit-type effect and can split the two fermionic helicity sectors near the horizon.


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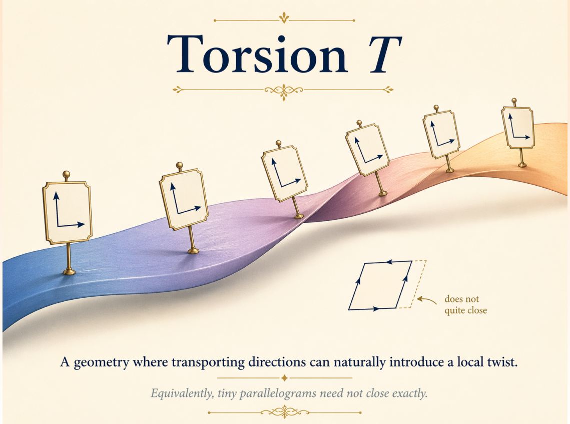

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